# On Broadcast in Generalized Network and Adversarial Models

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7th December 2020

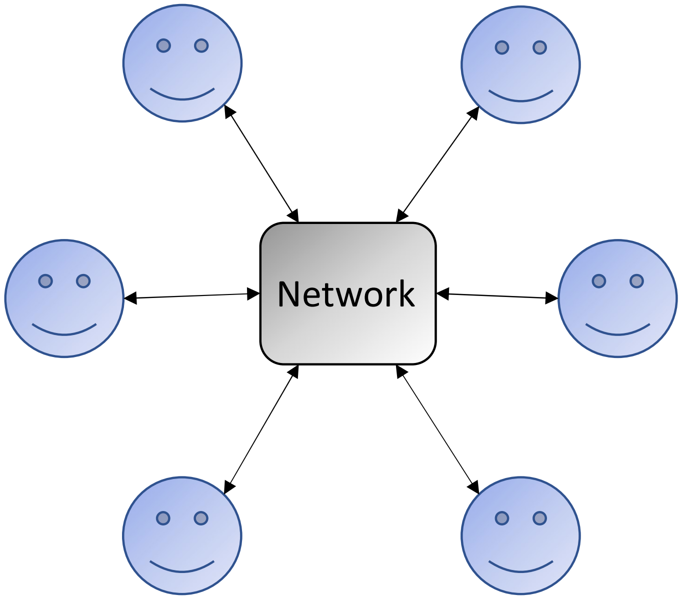
Joint work with Chen-Da Liu-Zhang and Ueli Maurer

Department of Computer Science, ETH Zurich

# Broadcast

## Setting

- Parties: $P = \{P_1, P_2, .., P_n\}$
- Synchronous
- No PKI setup



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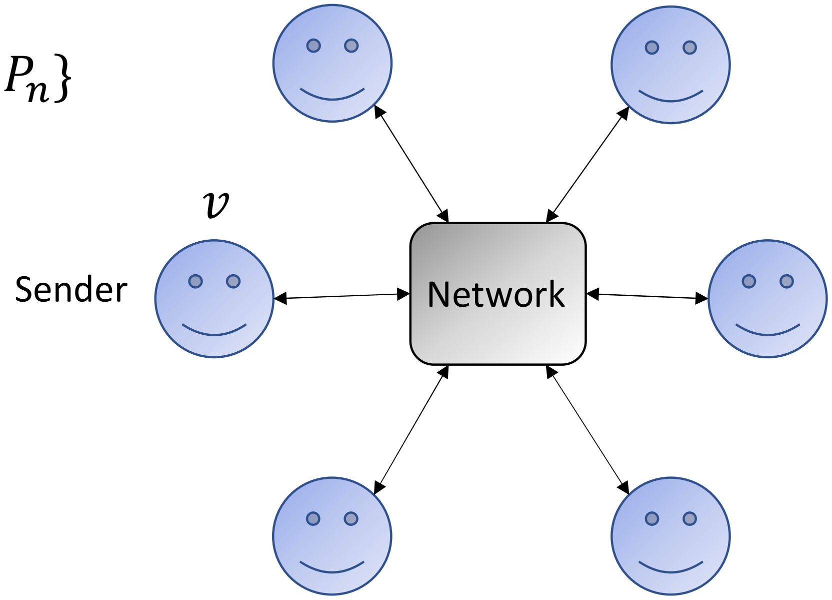

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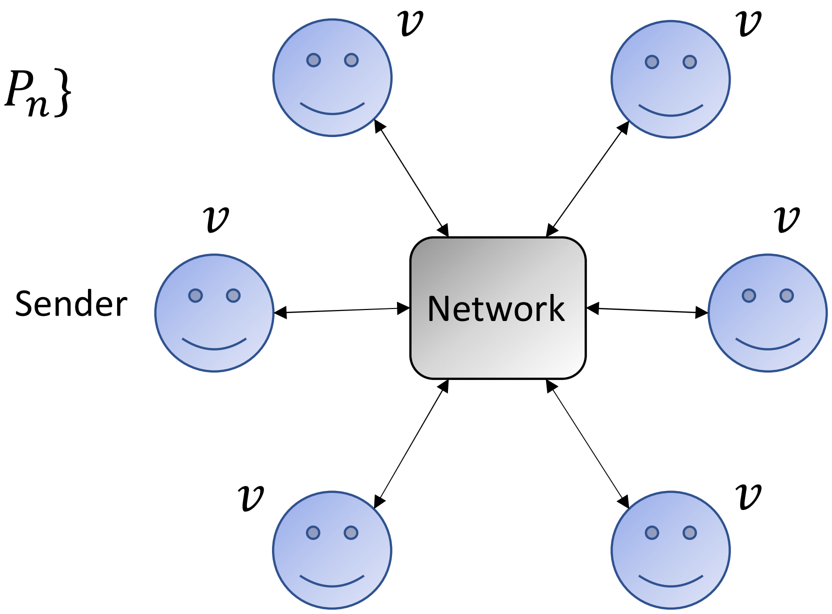

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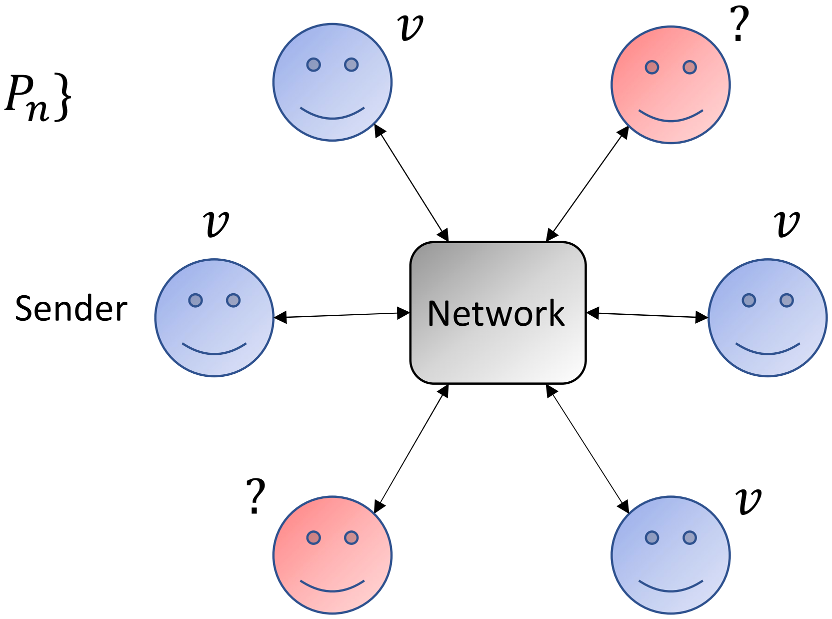
## Adversary

- Static
- Active (Byzantine)
- Unbounded



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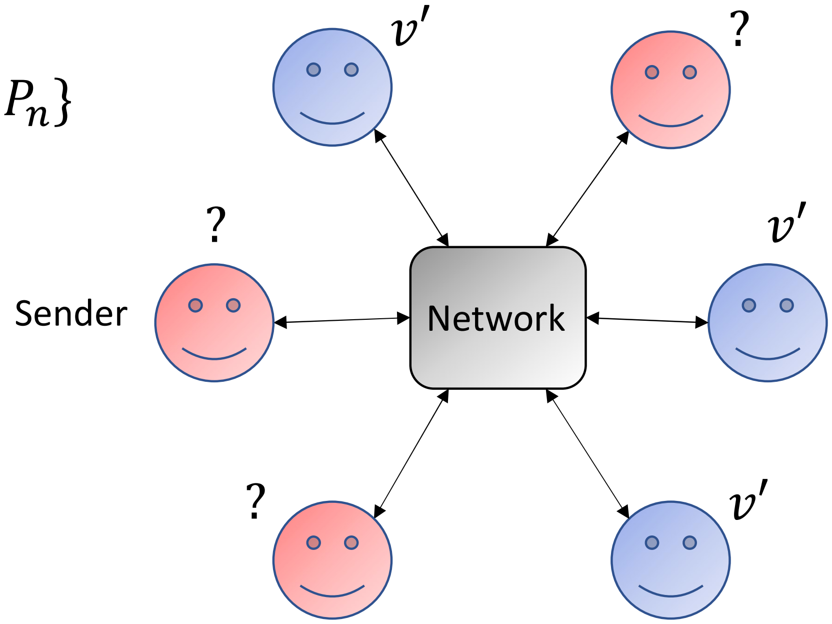
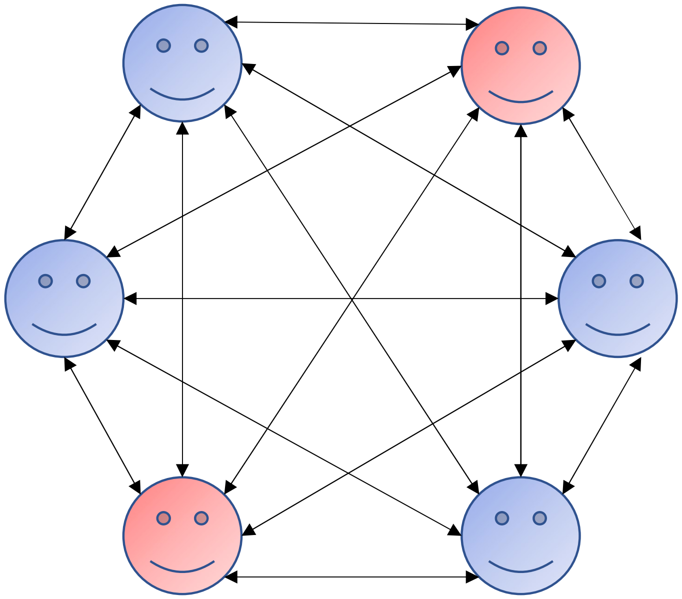


# Classical model

[PSL80]:

“Broadcast possible if and only if $t < n/3$”



# Classical model

[PSL80]:

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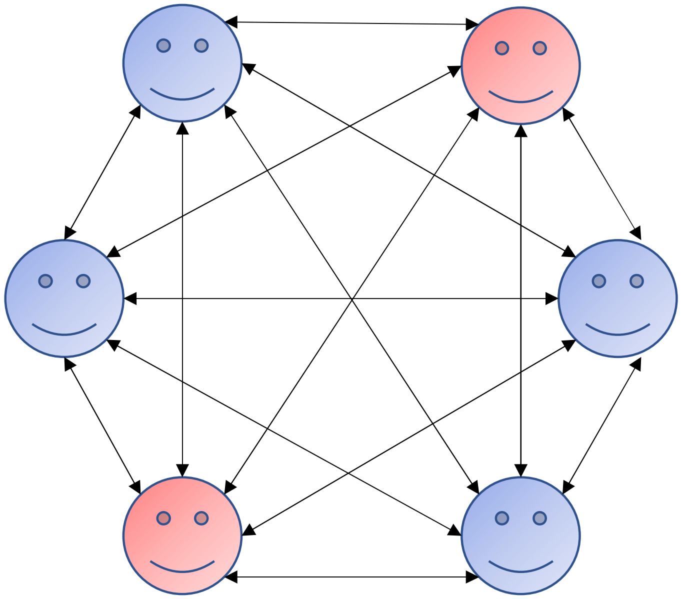
*Can we tolerate more?*



# 3-minicast model

[FM00]:

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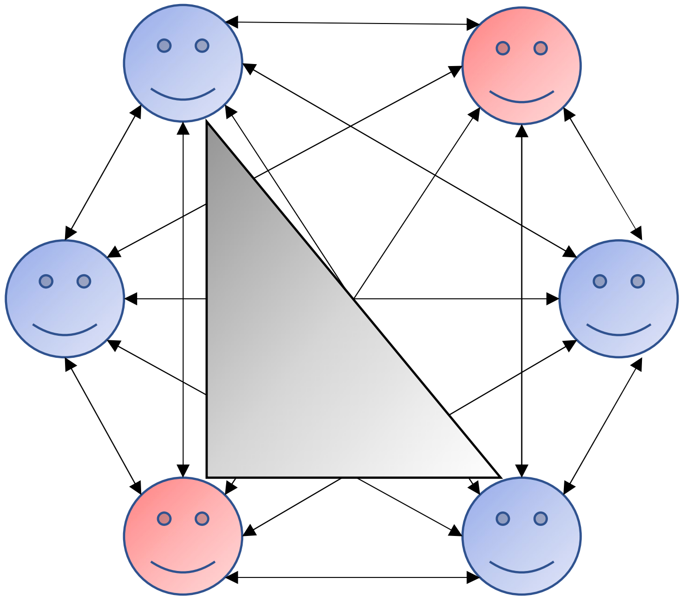

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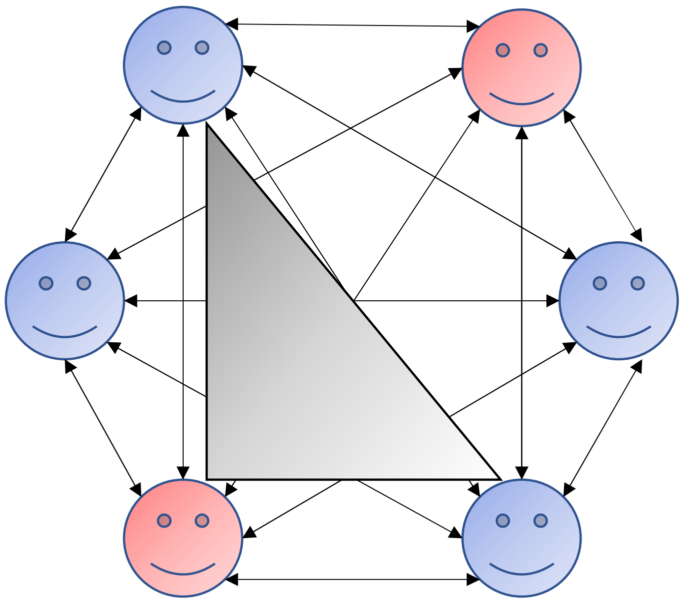
*Trade-off b/w power of network and power of adversary?*



# Current literature

| Adversary structure | Network structure | Reference |
|---|---|---|
| $t < n/3$ | Bilateral channels | [PSL80] |
| $t < n/2$ | 3-minicast channels | [FM00] |

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| $Q^{(3)}$ | Bilateral channels | [FM98] |

Threshold:
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General:
$A = \{A_1, A_2, .., A_k\}$
$A_i \subseteq P$

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*What's left to be done?*

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Complete!

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# General networks

Threshold:
$t < T$

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|---|---|---|
| $n/3 \leq t < n/2$ | Some 3-minicast channels | [RVS+04], [JMS12] |

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| Threshold:<br>$t < T$ | $n/3 \leq t < n/2$ | Some 3-minicast channels | [RVS$^{+}$04], [JMS12] |
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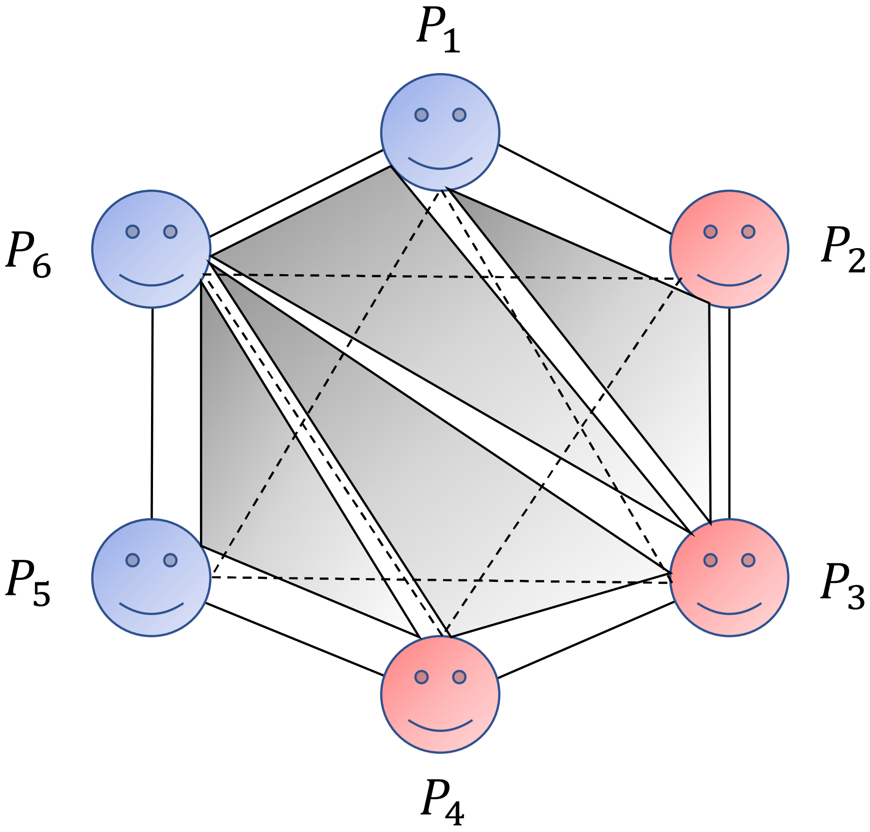
| General:<br>$A = \{A_1, A_2, .., A_k\}$<br>$A_i \subseteq P$ | $A$ contains $b$-chain(s) and $A$ is $(b + 1)$-chain free $\left(A \in \mathfrak{A}^{(b)}\right)$ | Some $b$-minicast channels | [LMM20] |

# Example

- 3-minicast model
- Six parties
- $t \leq 3$



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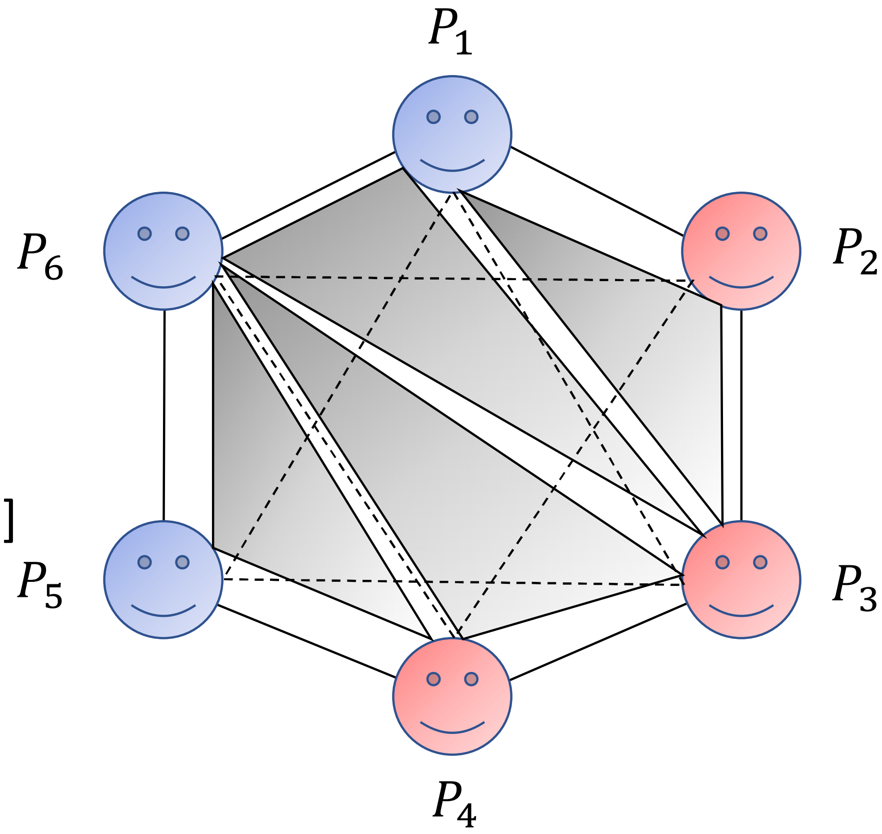

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- 4-minicast model
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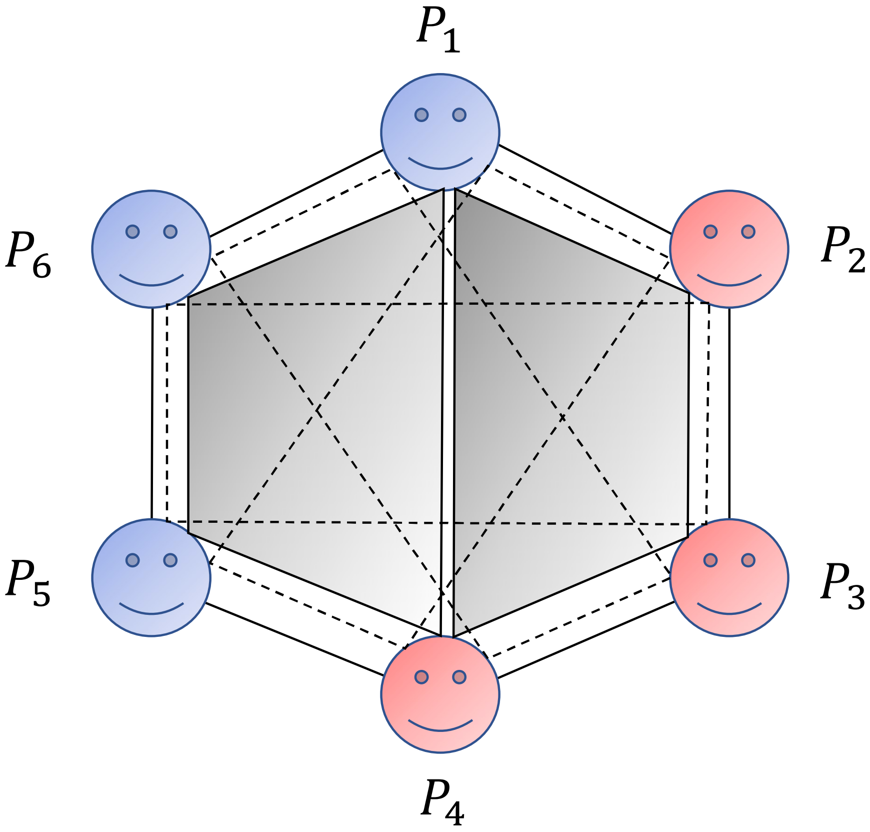

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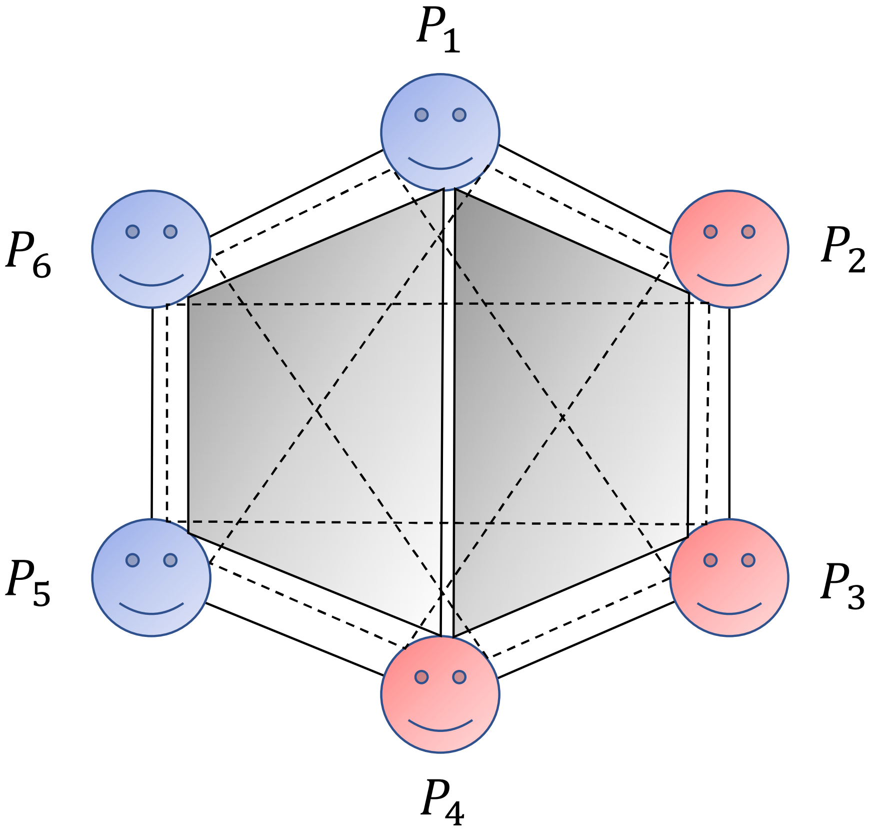

# Example

- Some 4-minicast channels
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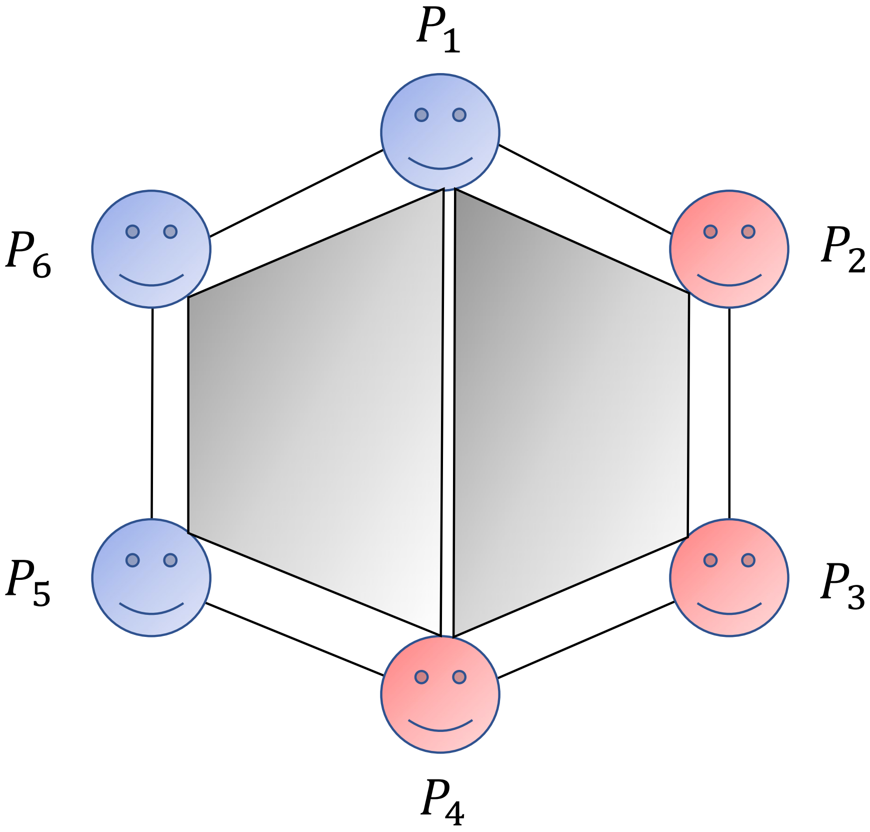

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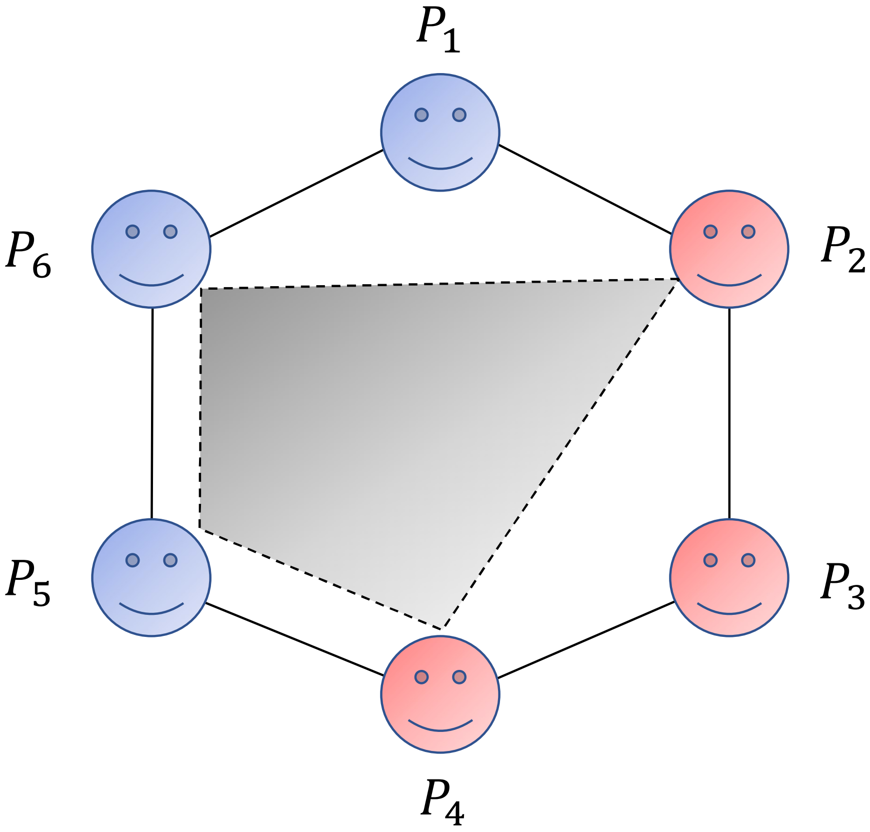

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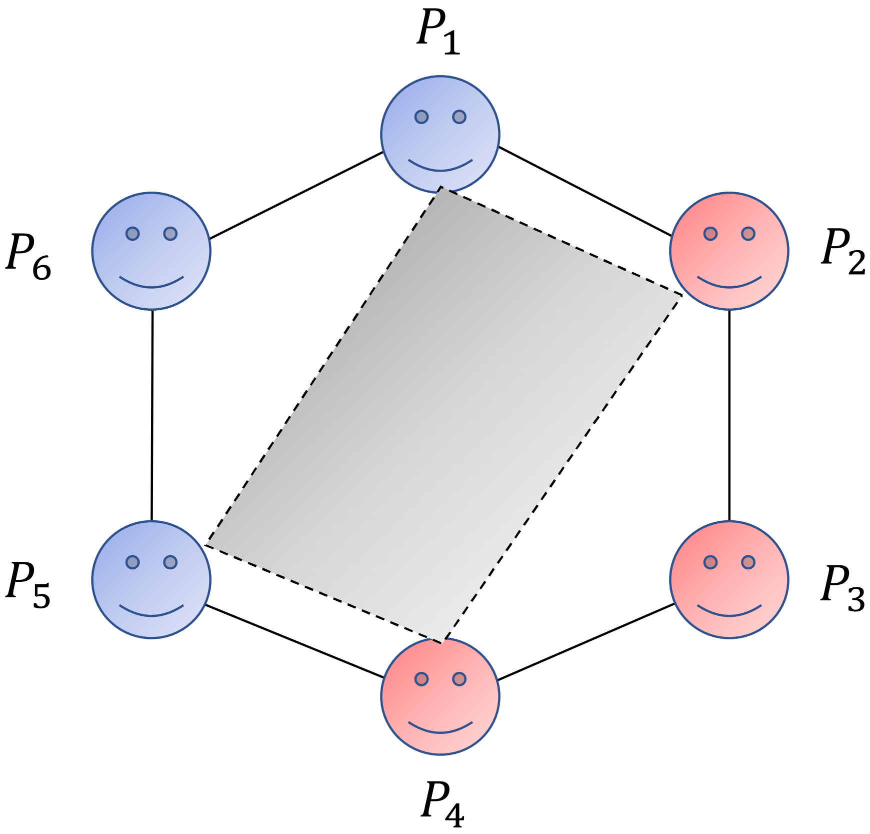

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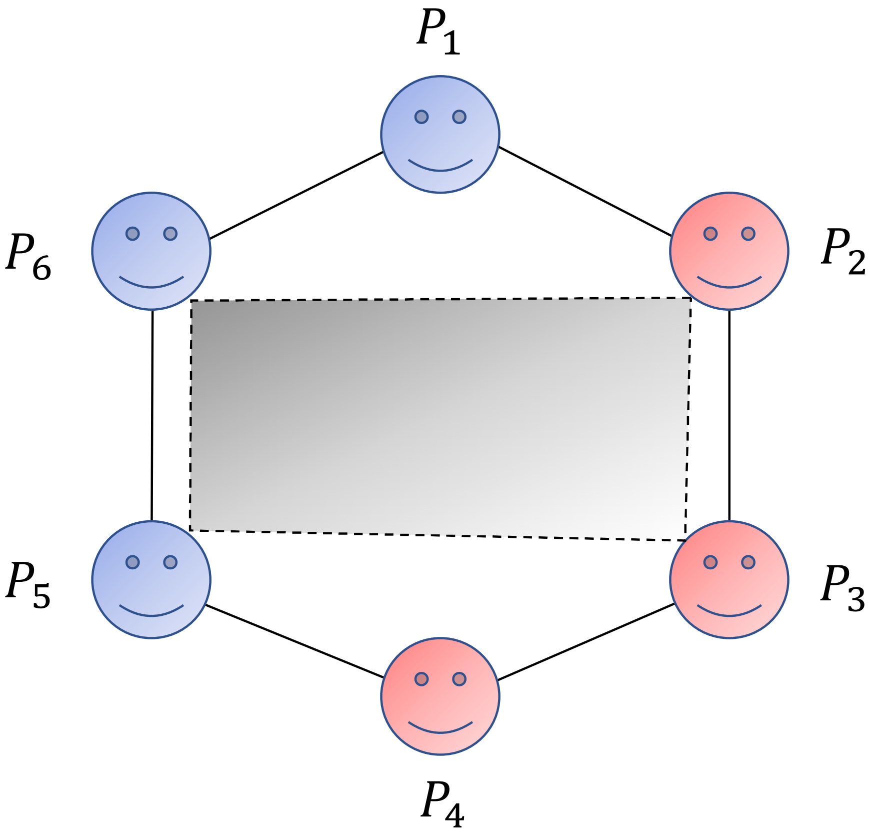

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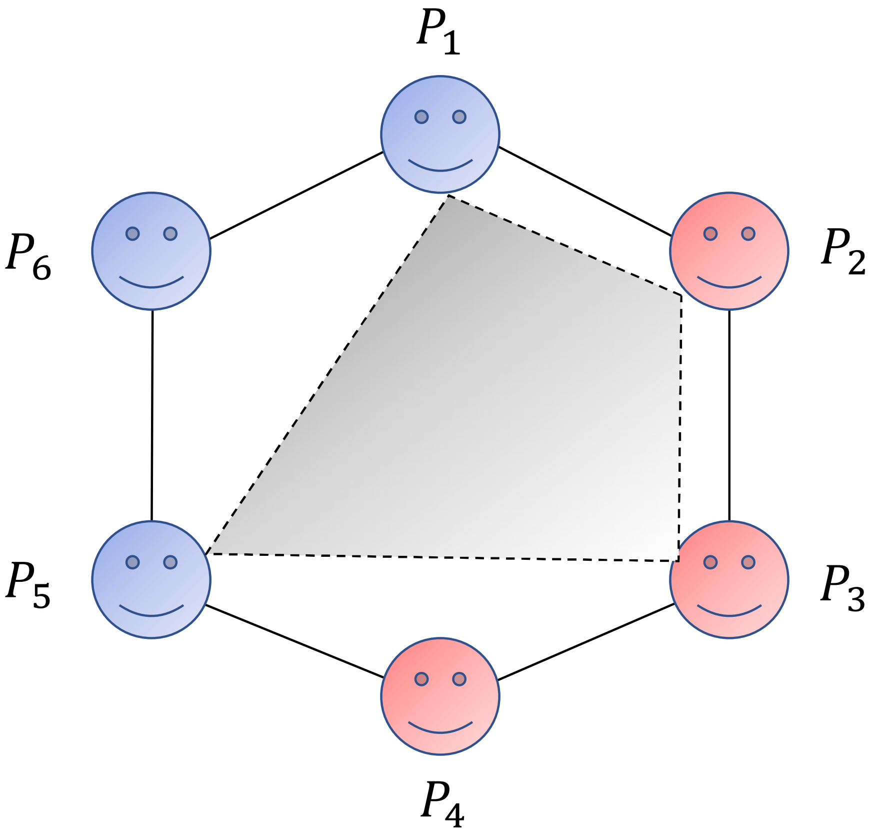

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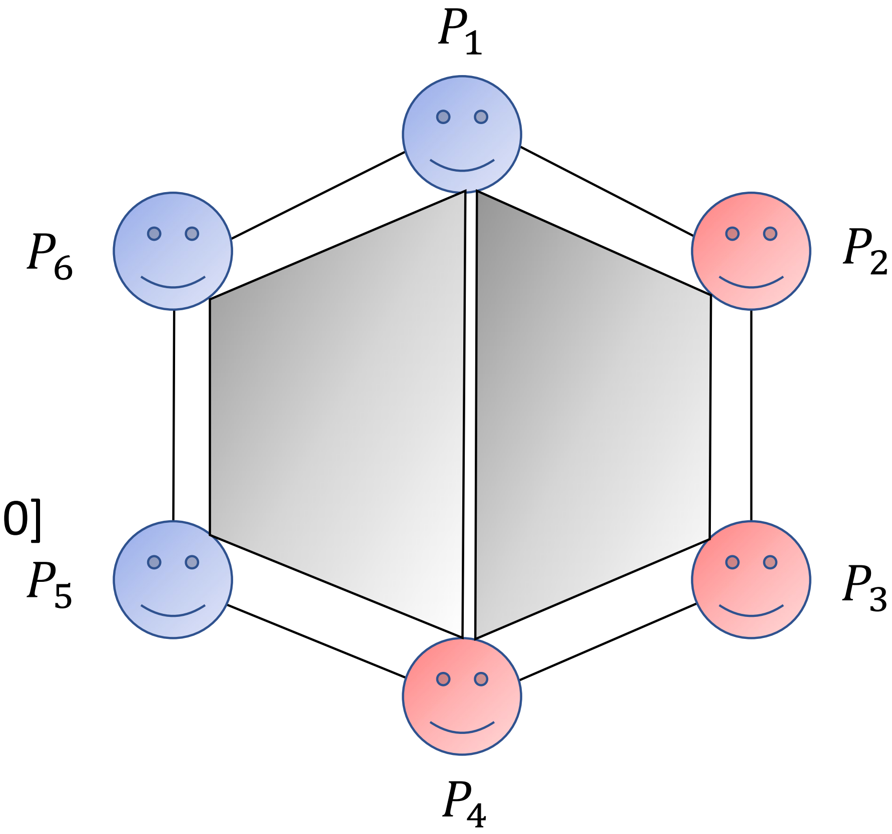

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- Some 4-minicast channels
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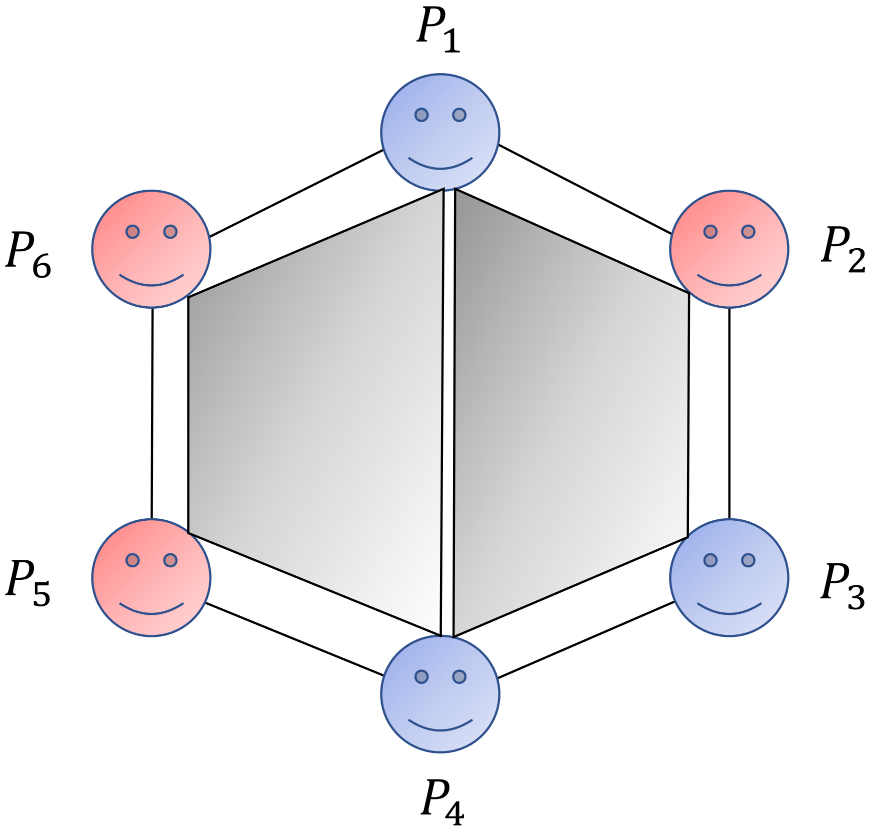
- $A = \{A_1, A_2, .., A_k\}$
  $(|A_i| = 3)$



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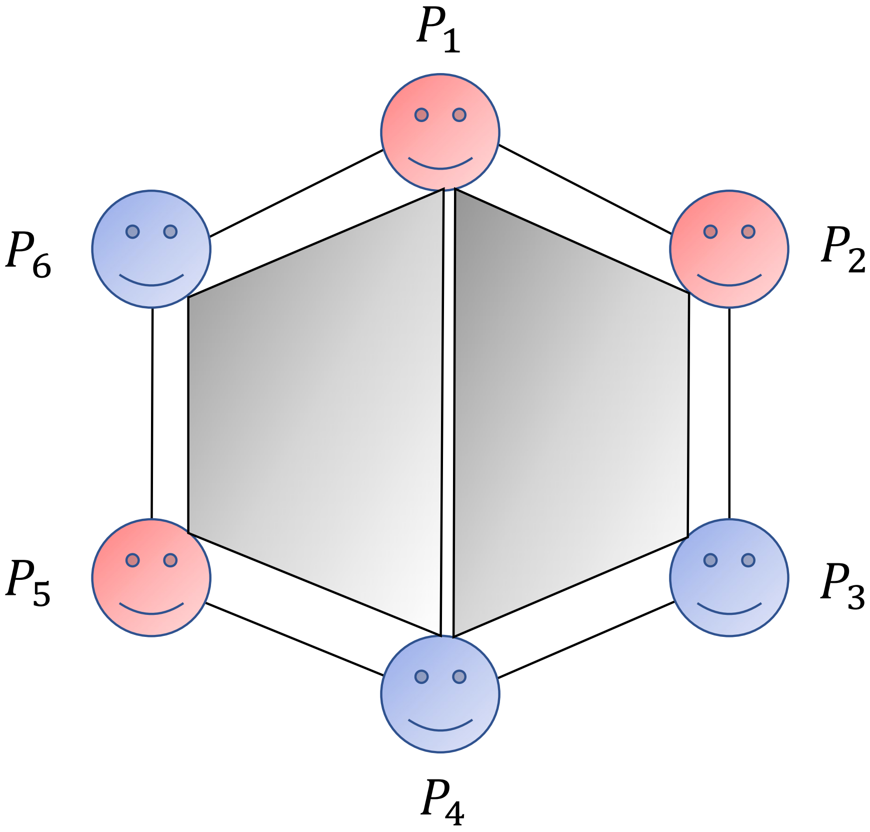

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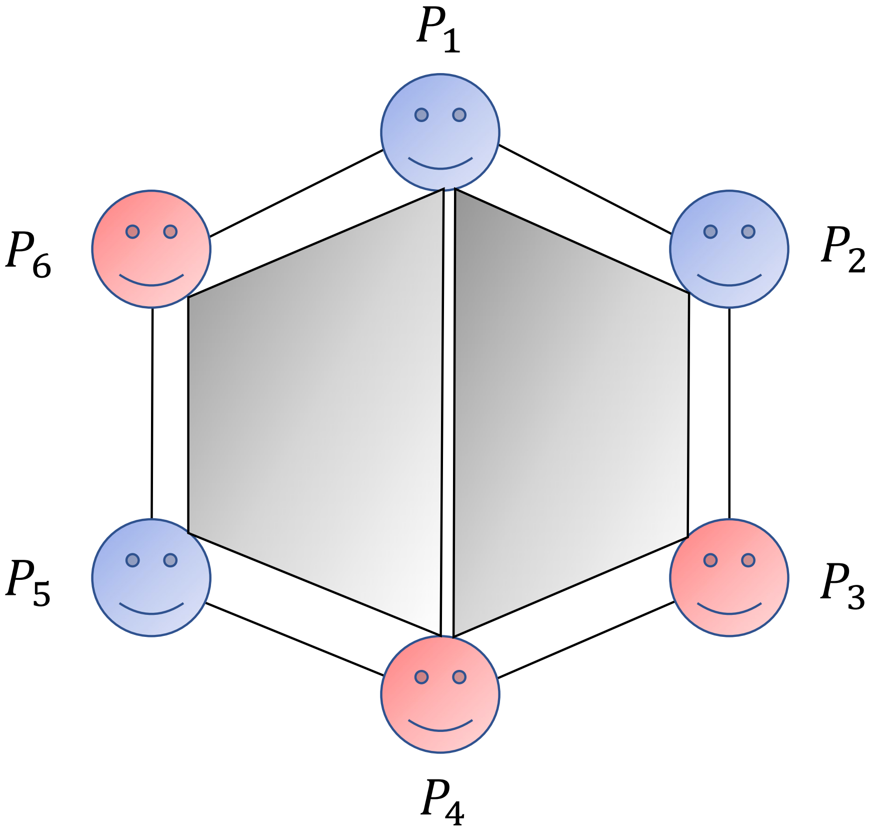

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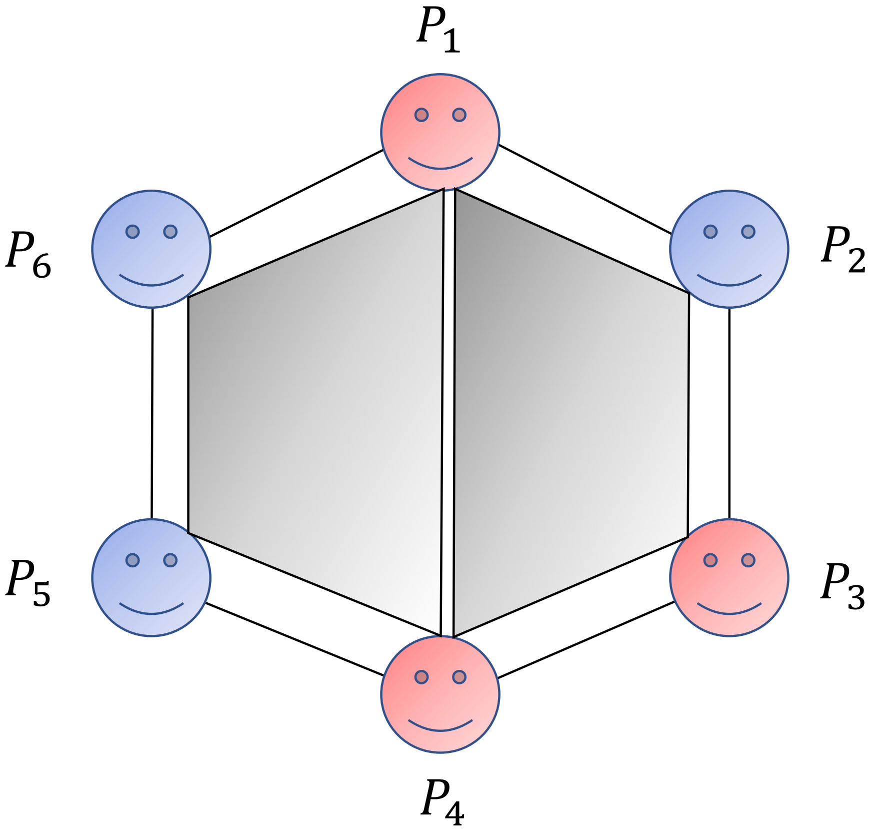

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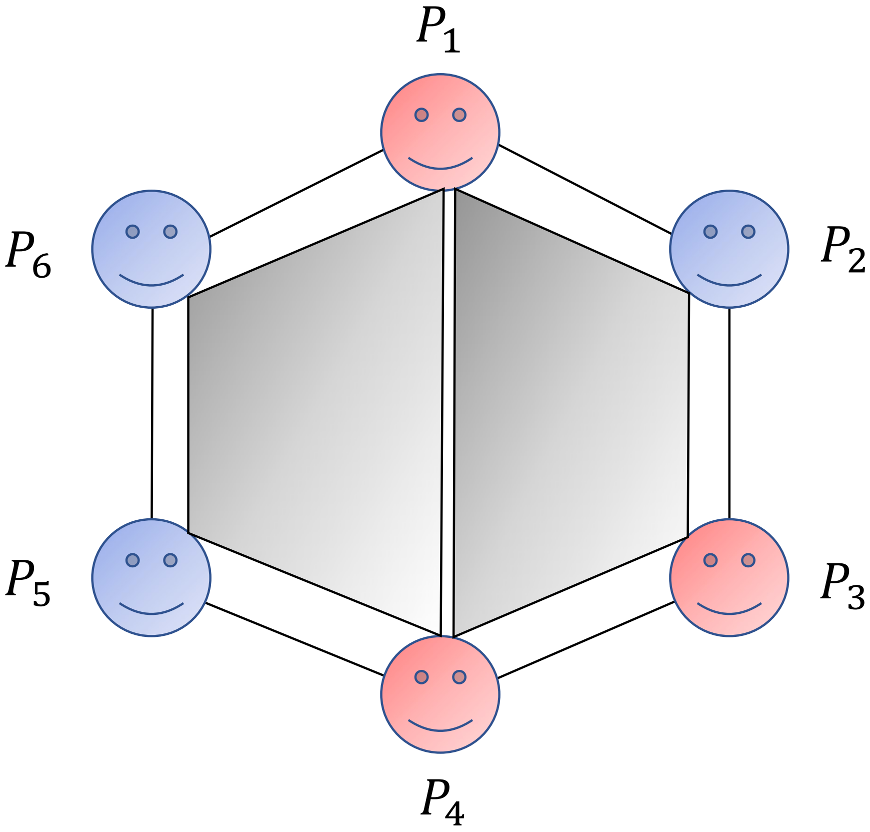

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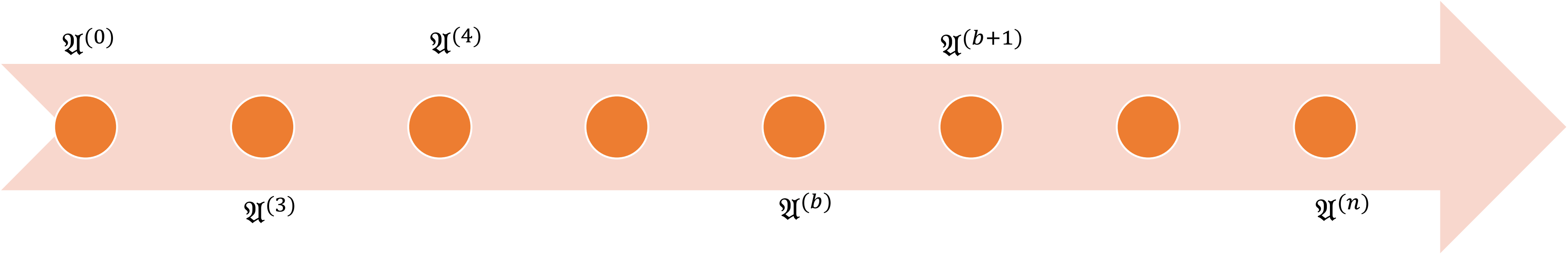

$\left(A \in \mathfrak{A}^{(0)}\right)$: $A$ is 3-chain free

$\left(A \in \mathfrak{A}^{(b)}\right)$: $A$ contains $b$-chain(s) and $A$ is $(b+1)$-chain free, for $3 \leq b \leq n$

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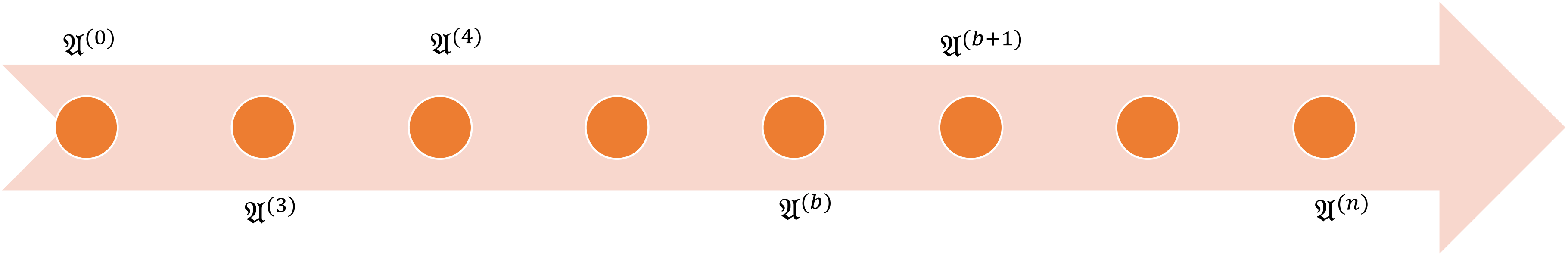

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$A$ is $b$-chain free $\Longrightarrow$ $A$ is $(b+1)$-chain free

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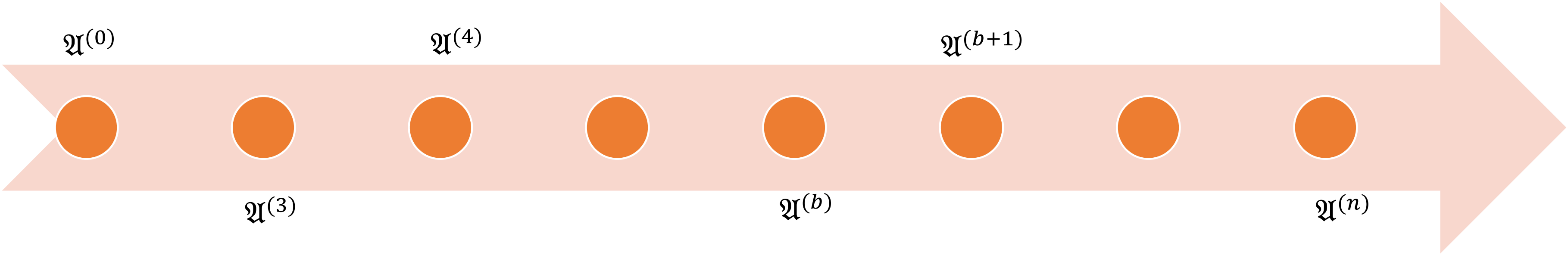

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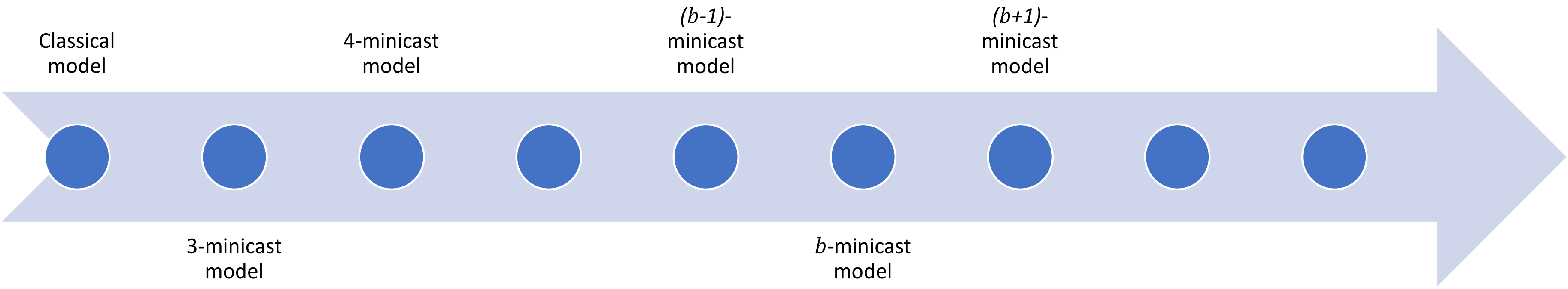
$\mathfrak{A}^{(3)}$

$\mathfrak{A}^{(4)}$

$\mathfrak{A}^{(b)}$

$\mathfrak{A}^{(b+1)}$

$\mathfrak{A}^{(n)}$



Classical model

3-minicast model

4-minicast model

*(b-1)*-minicast model

*b*-minicast model

*(b+1)*-minicast model

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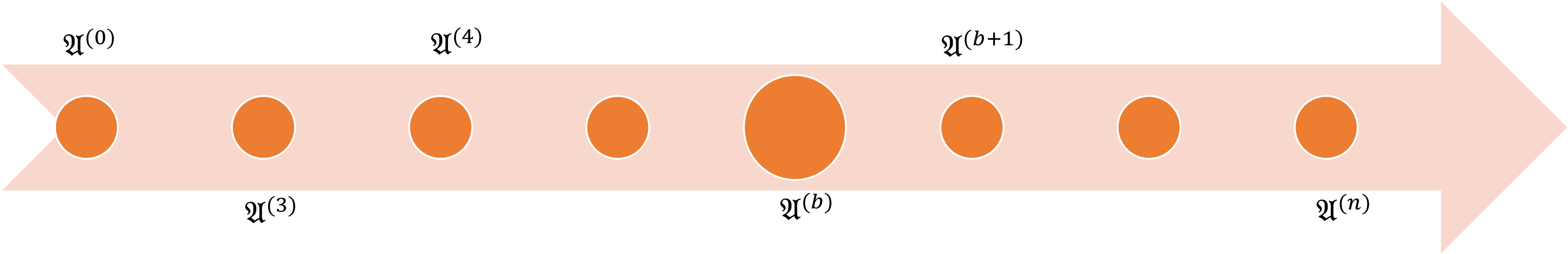

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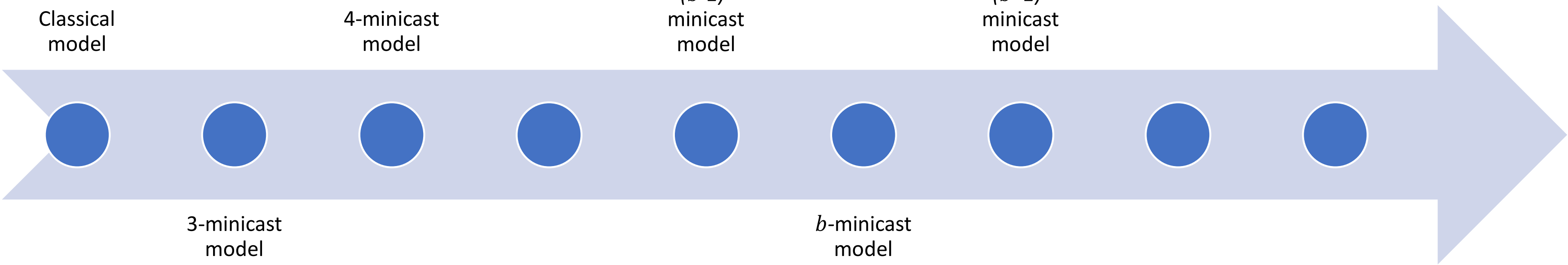

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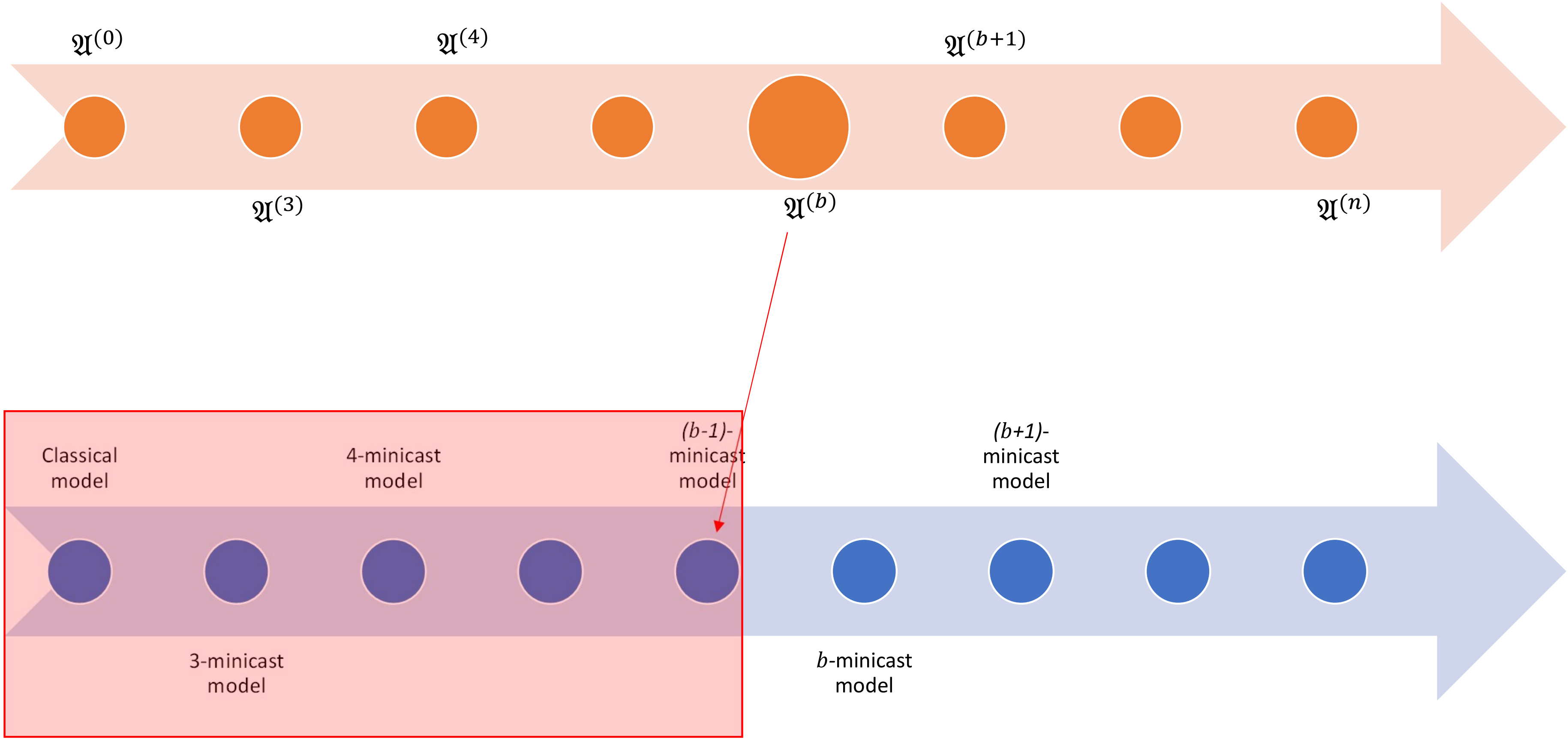
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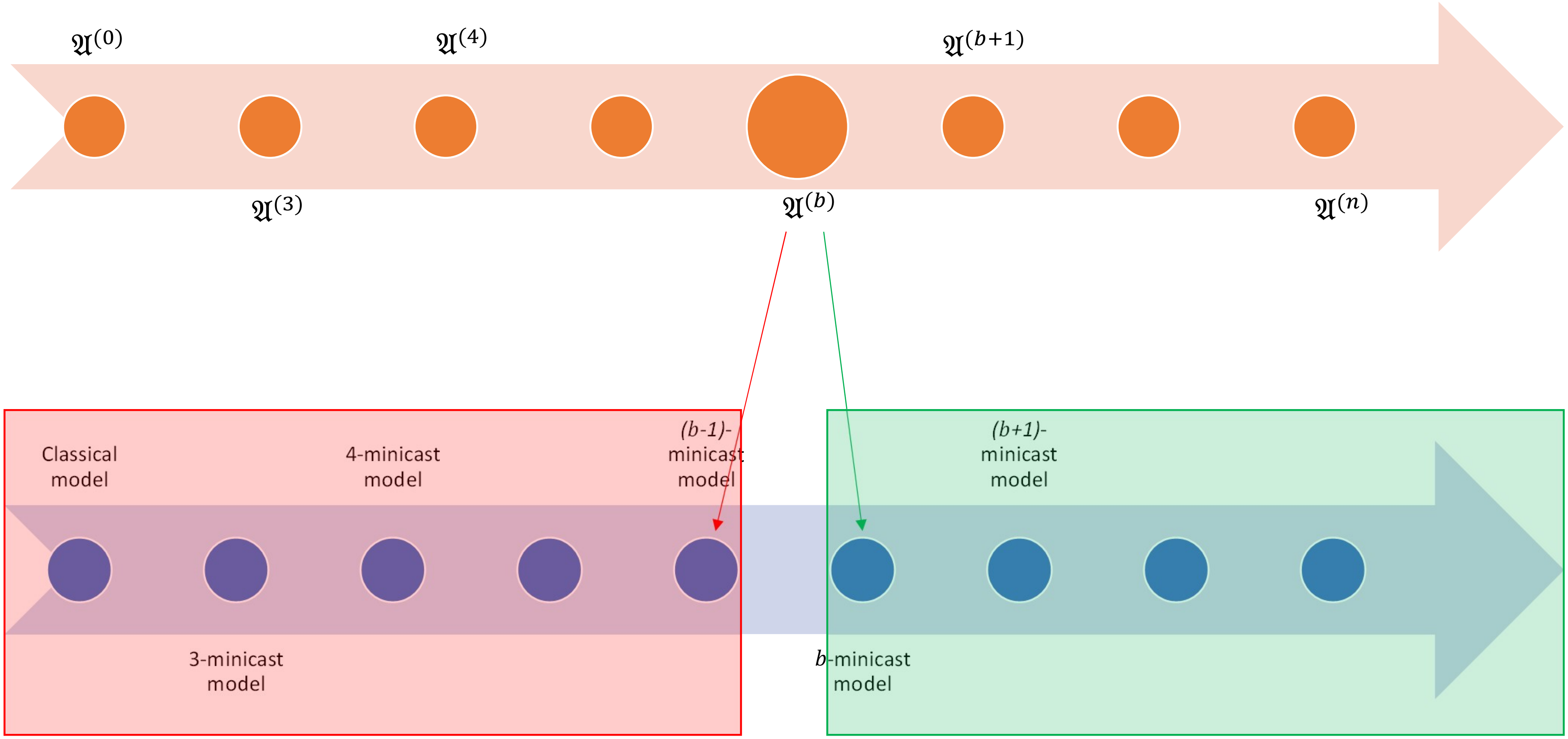
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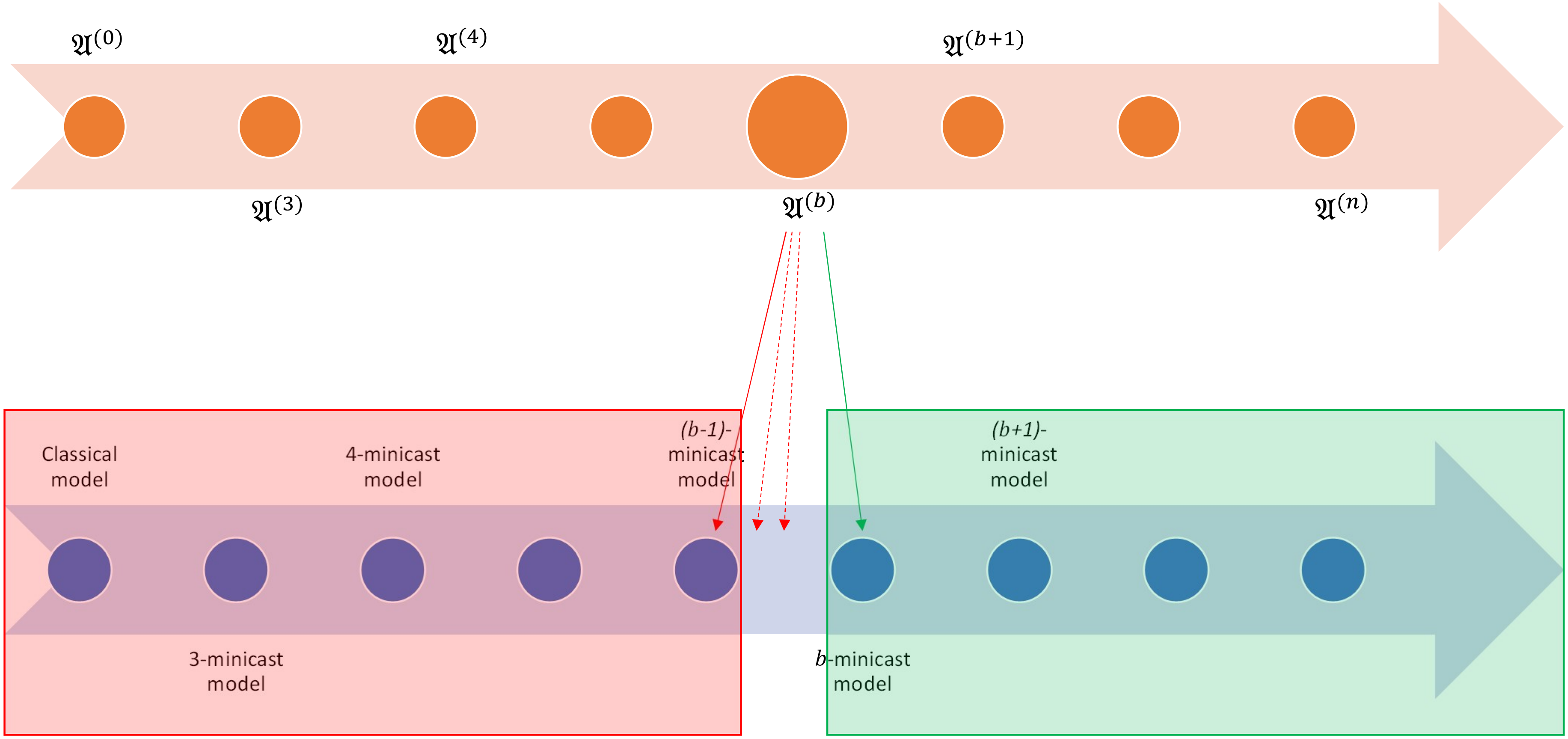

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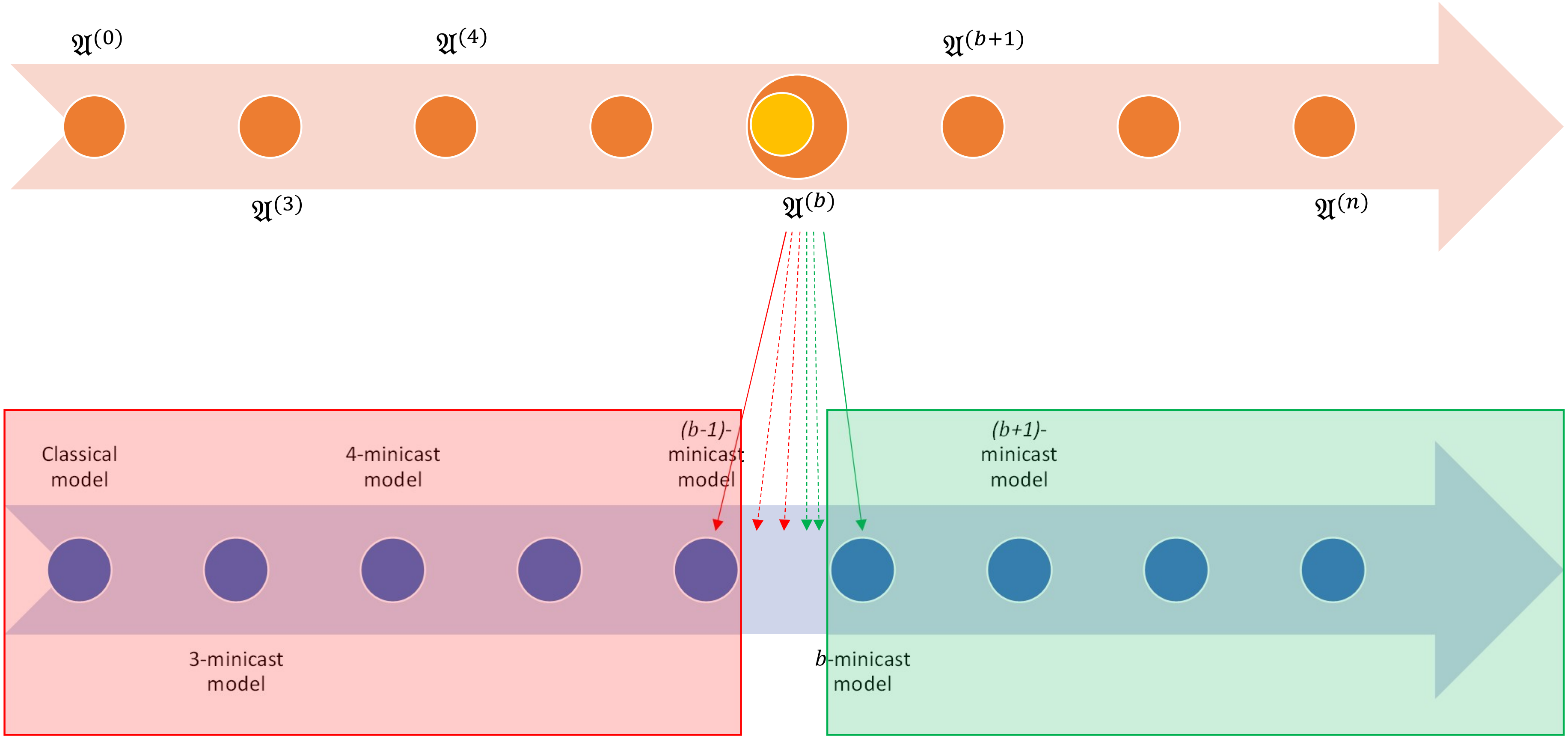
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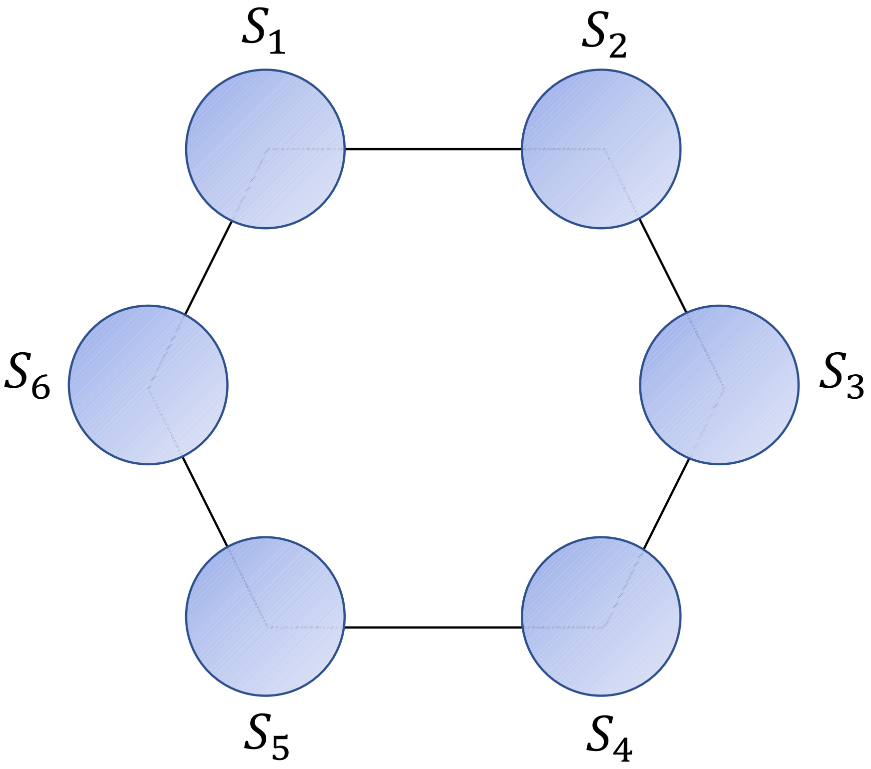

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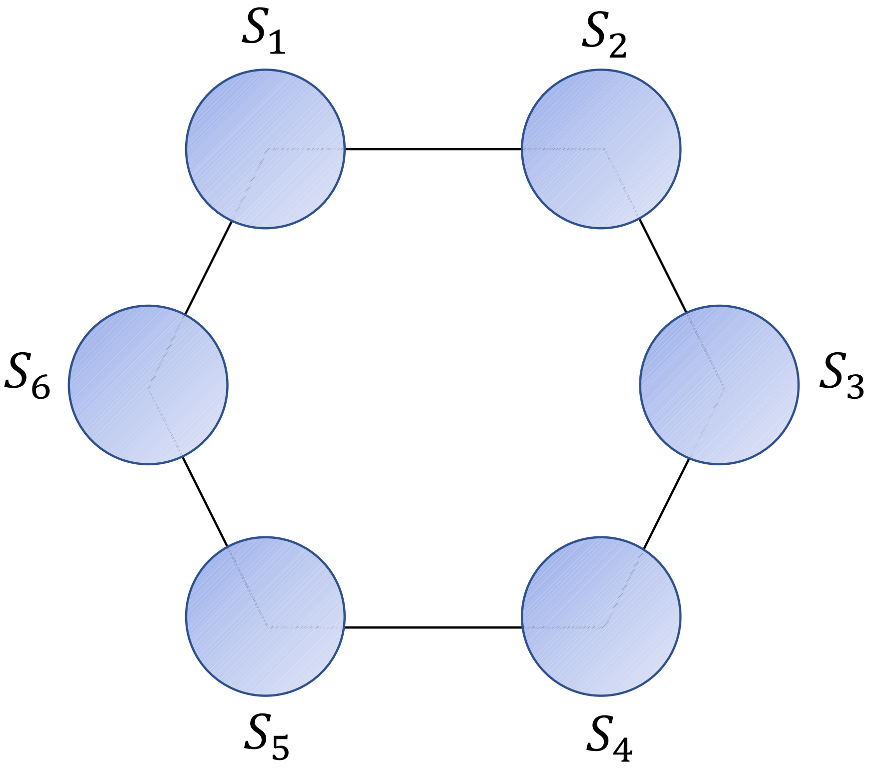
- Partition: $S = (S_1, S_2, .., S_b)$
  - $\cup_{i=1}^{b} S_i = P$
  - $S_i \cap S_j = \emptyset$
  - non-empty $S_i$'s



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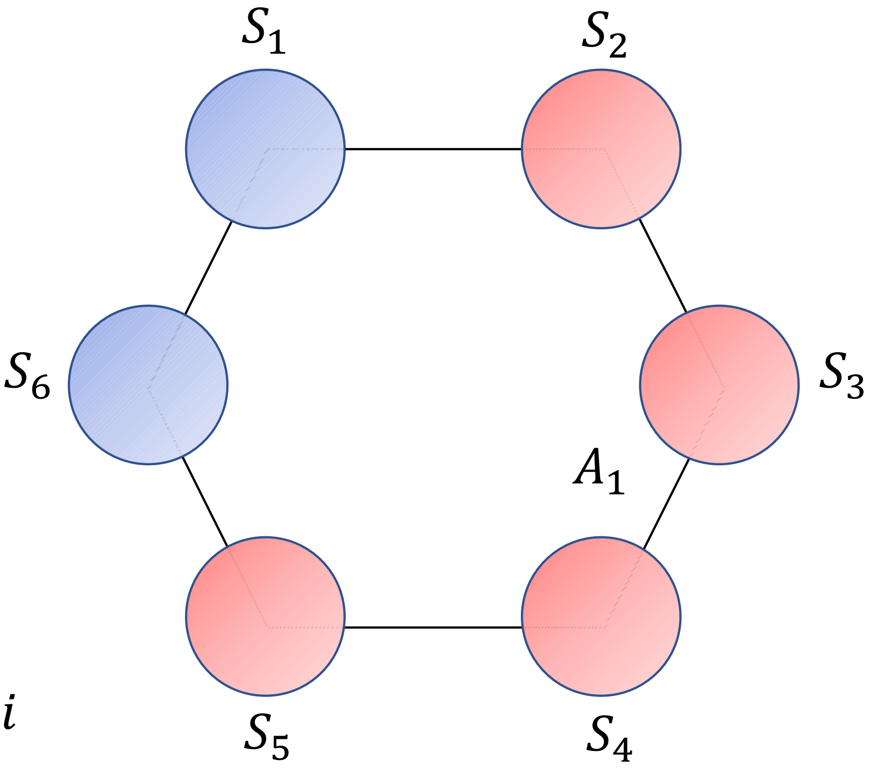

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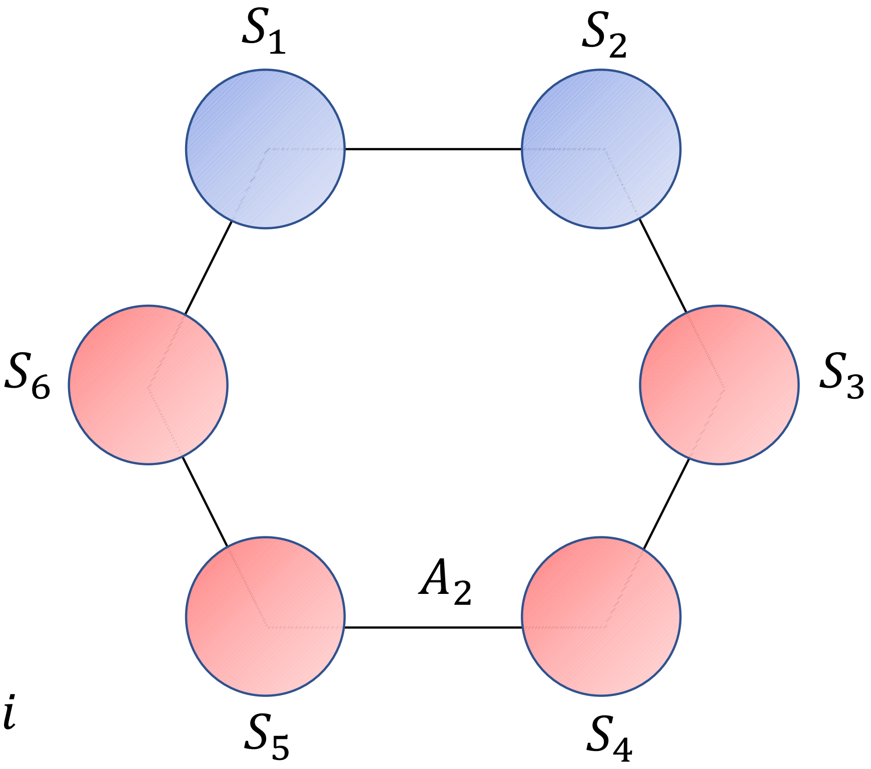

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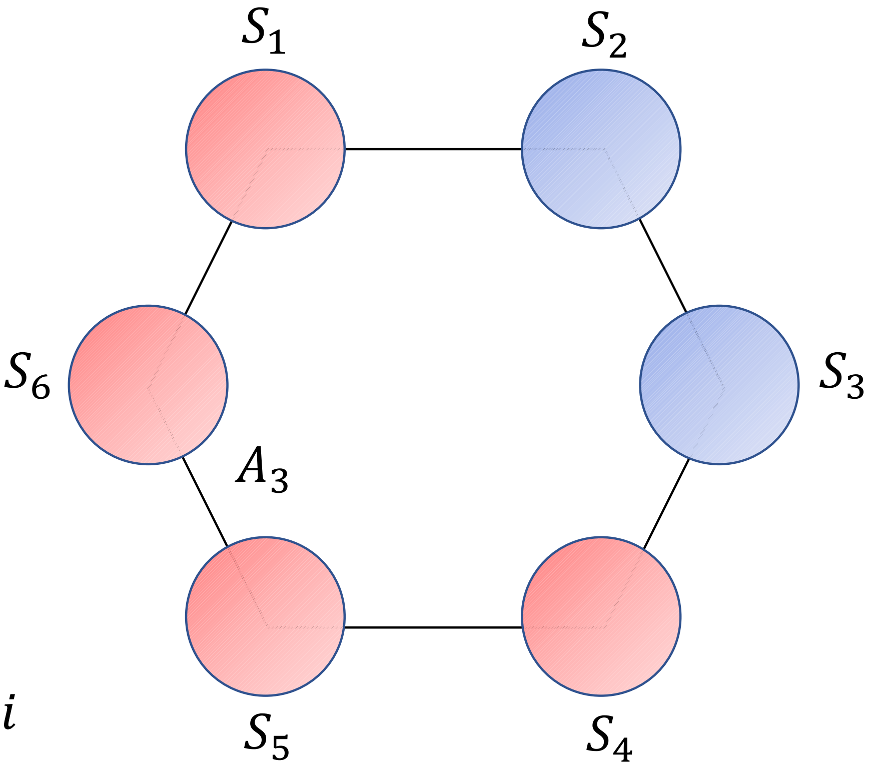

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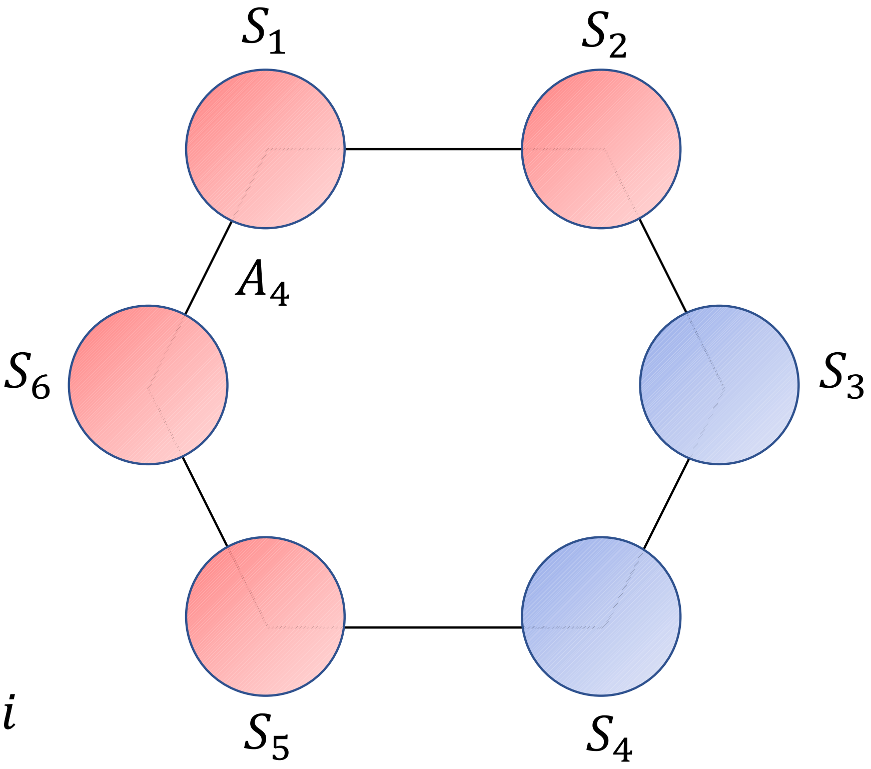

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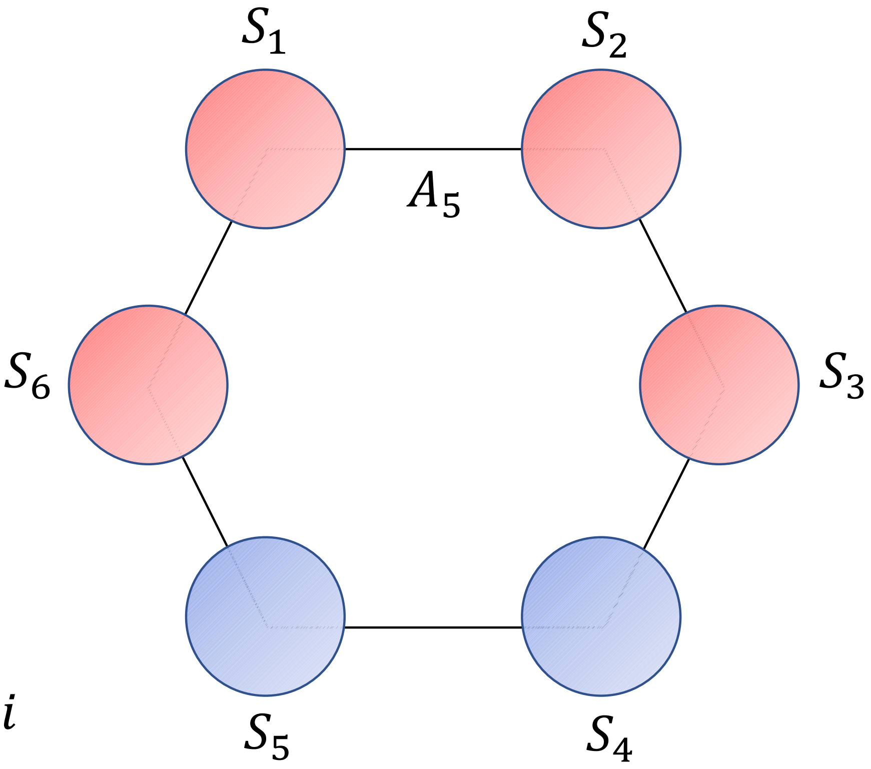

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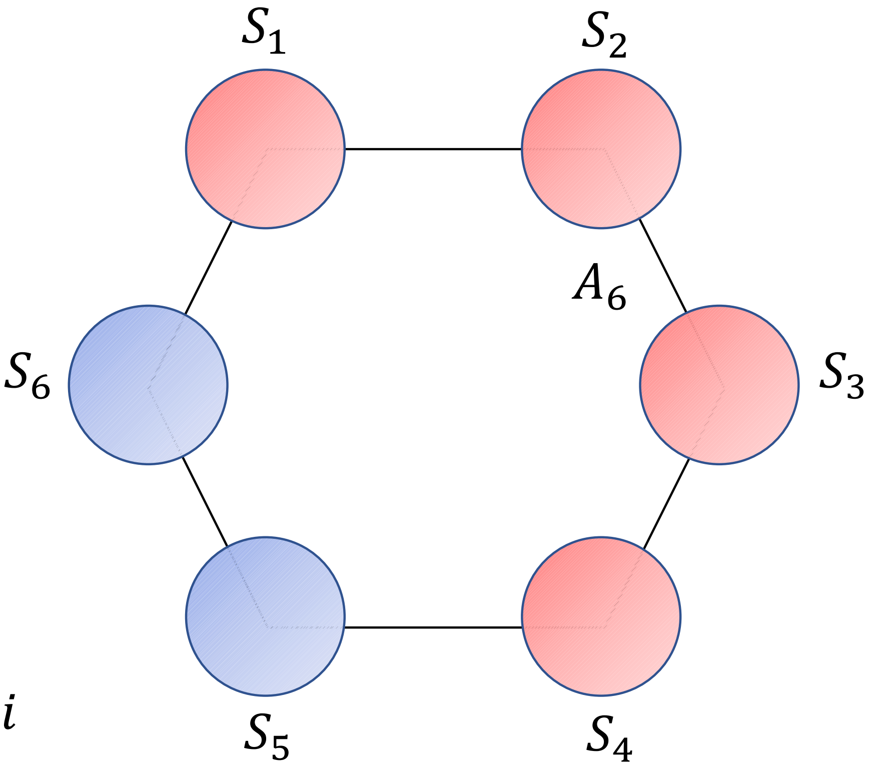

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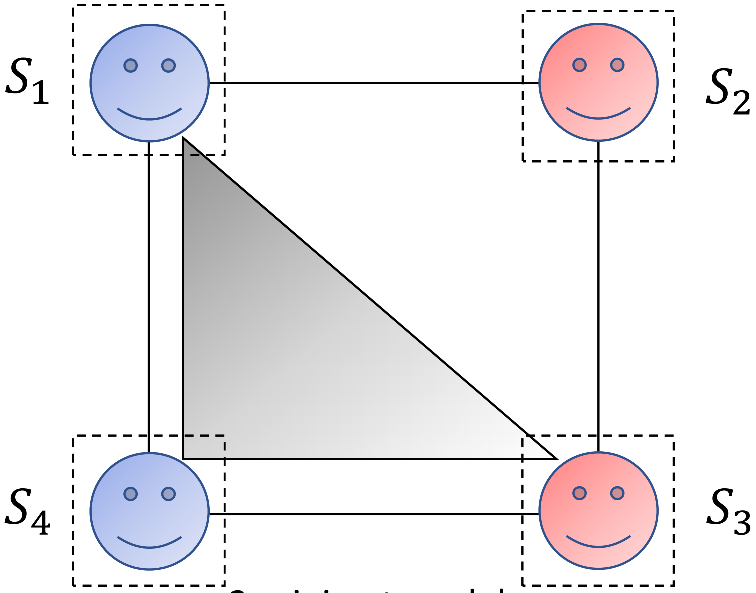

3-minicast model
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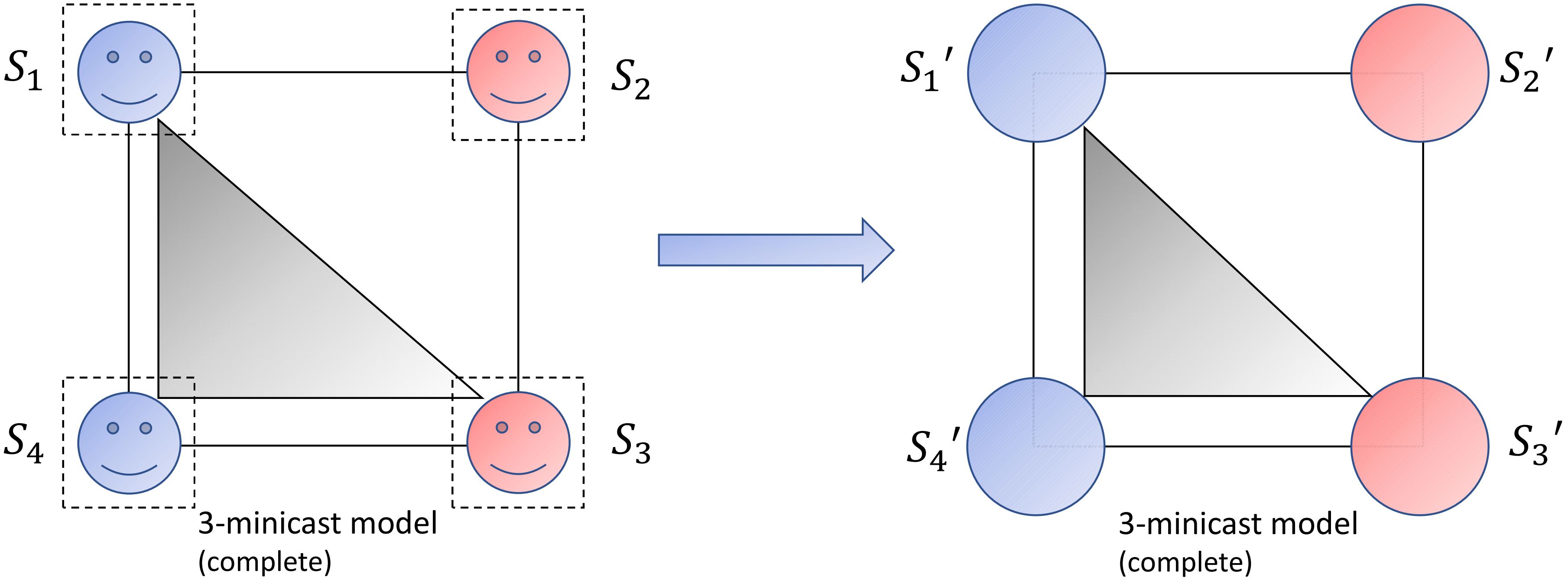

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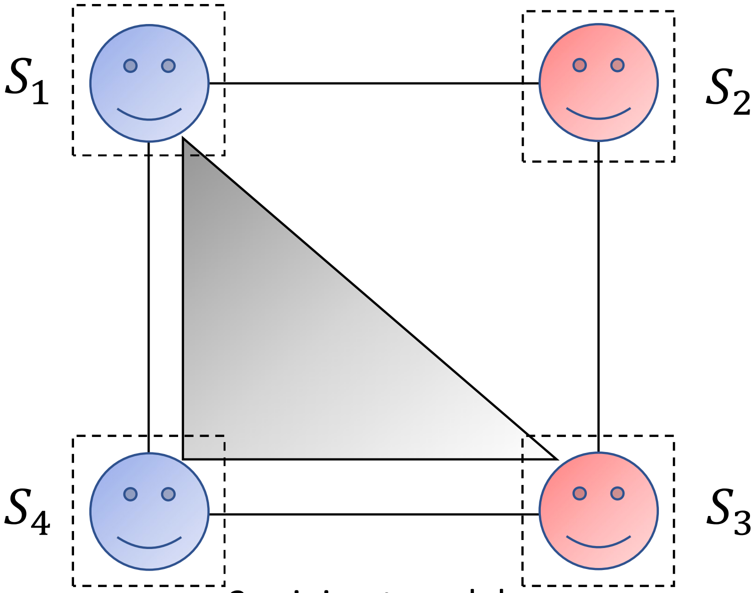

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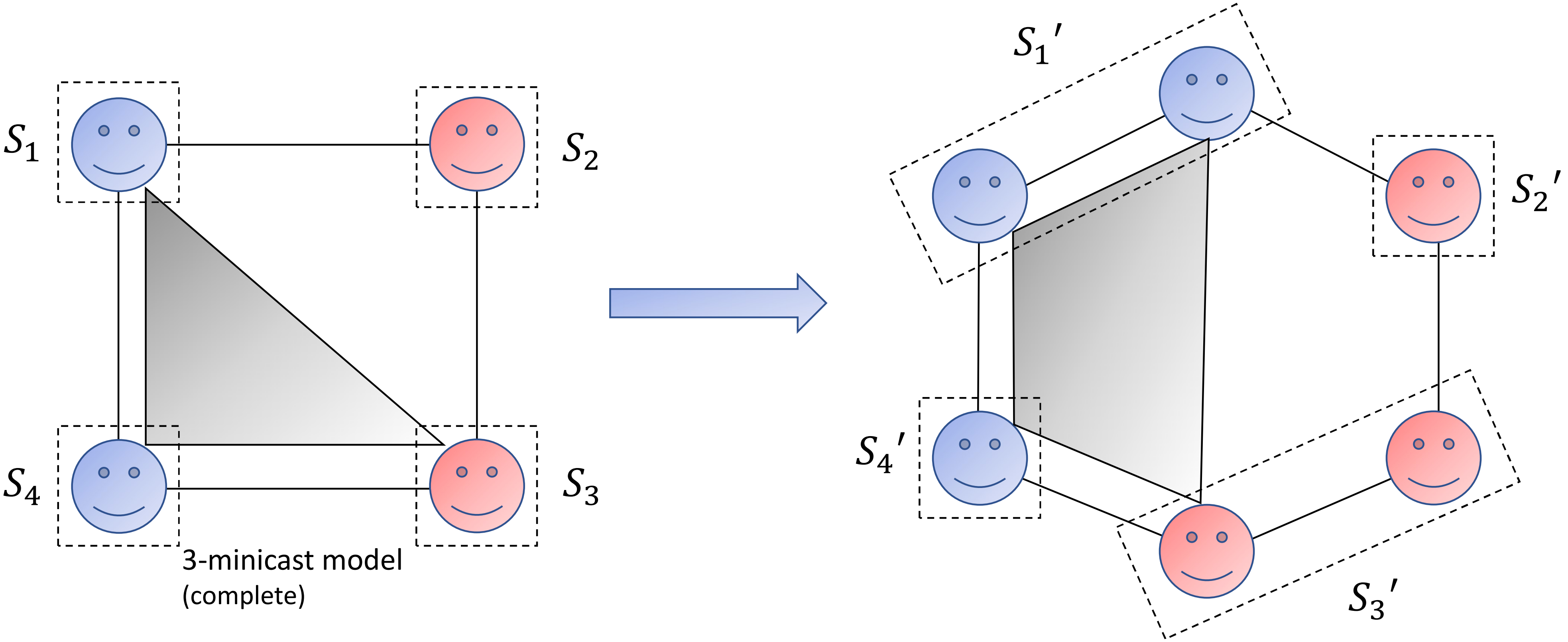

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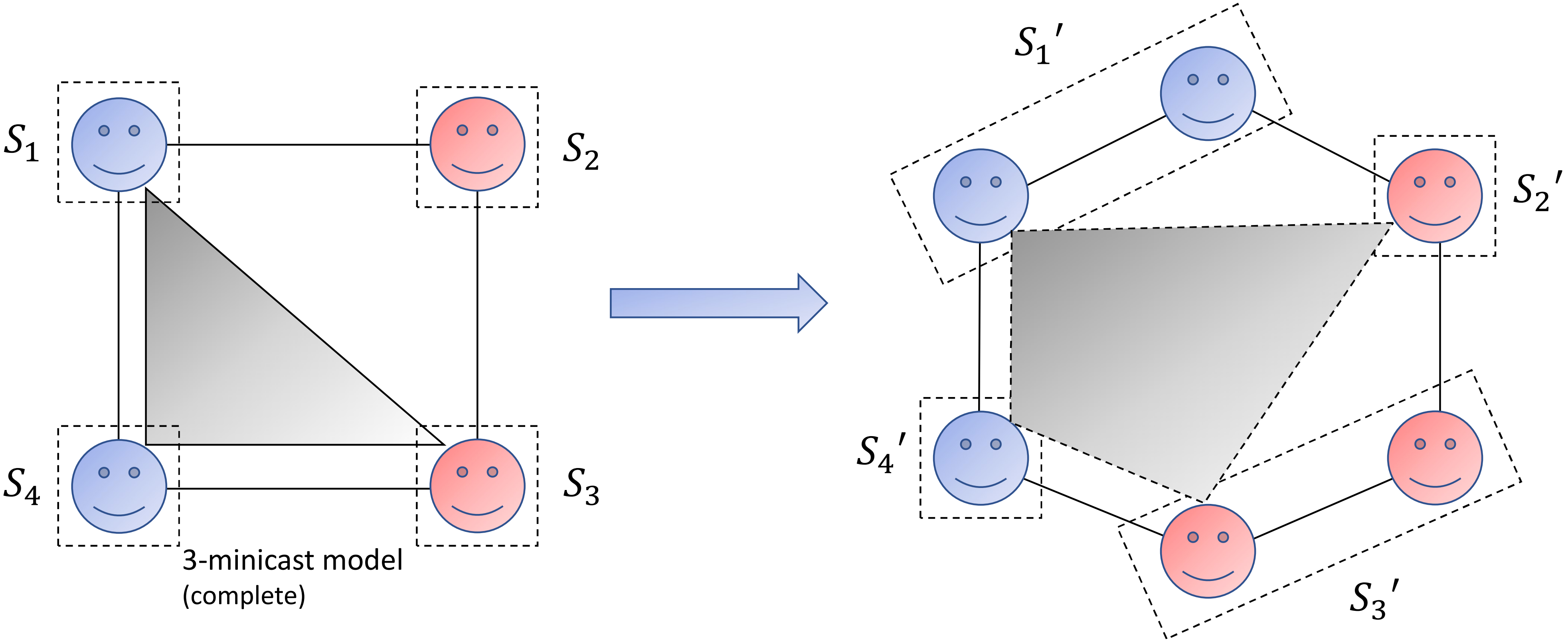

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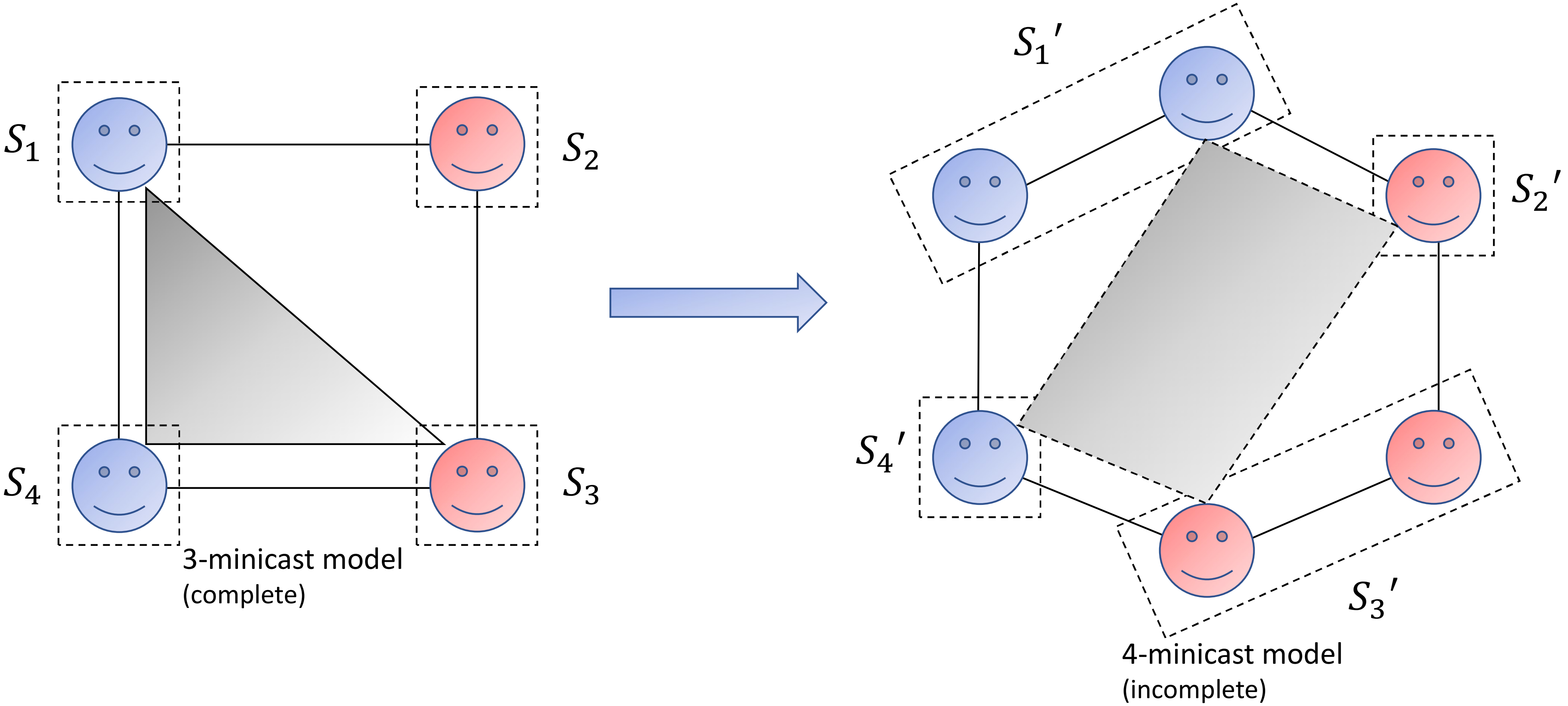

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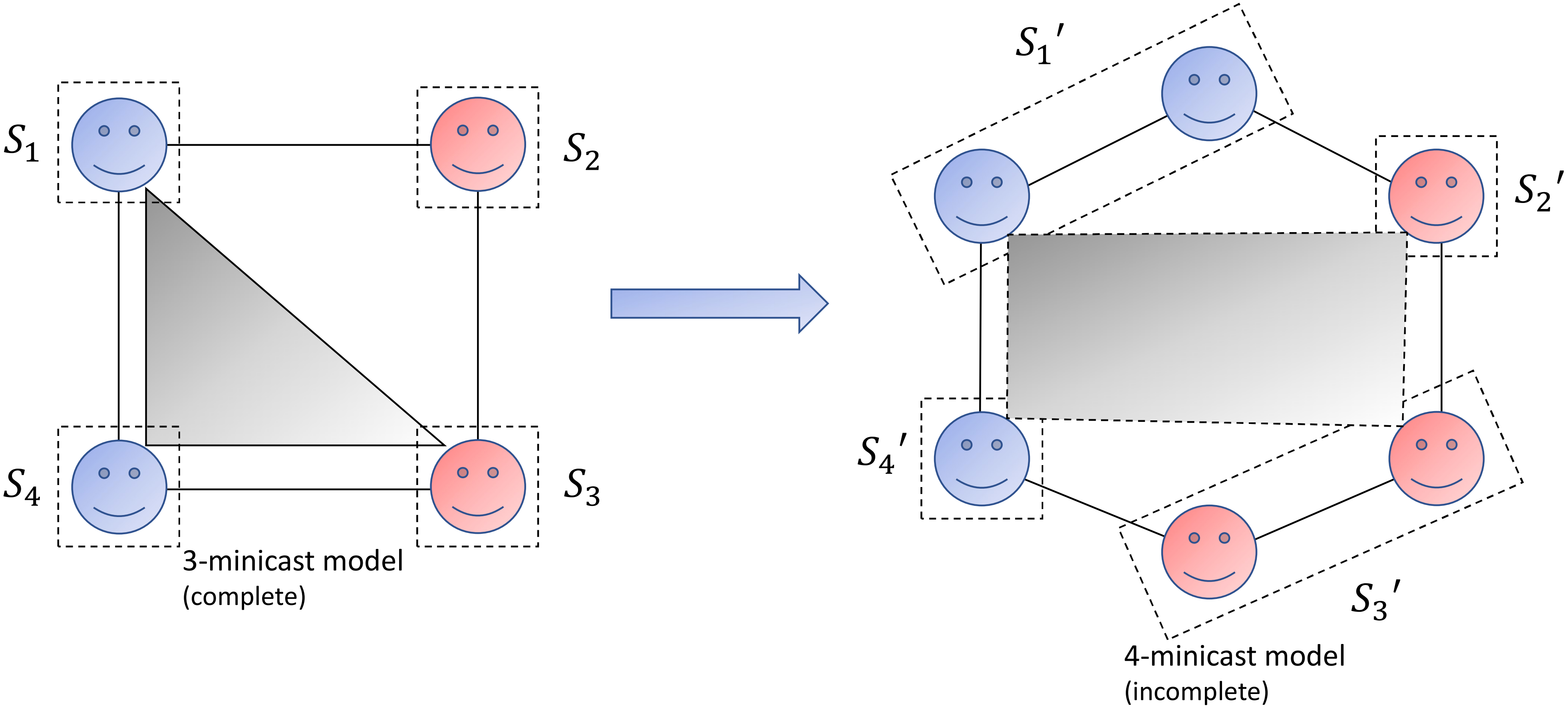

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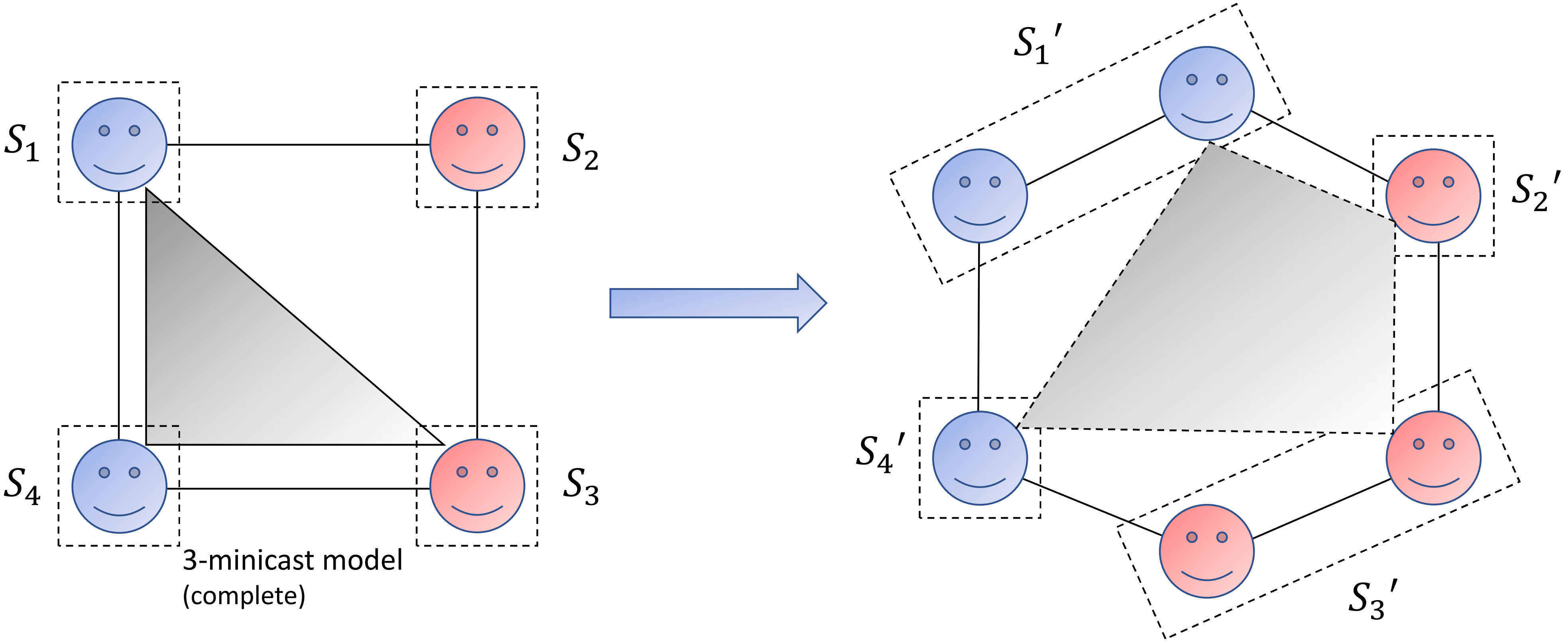

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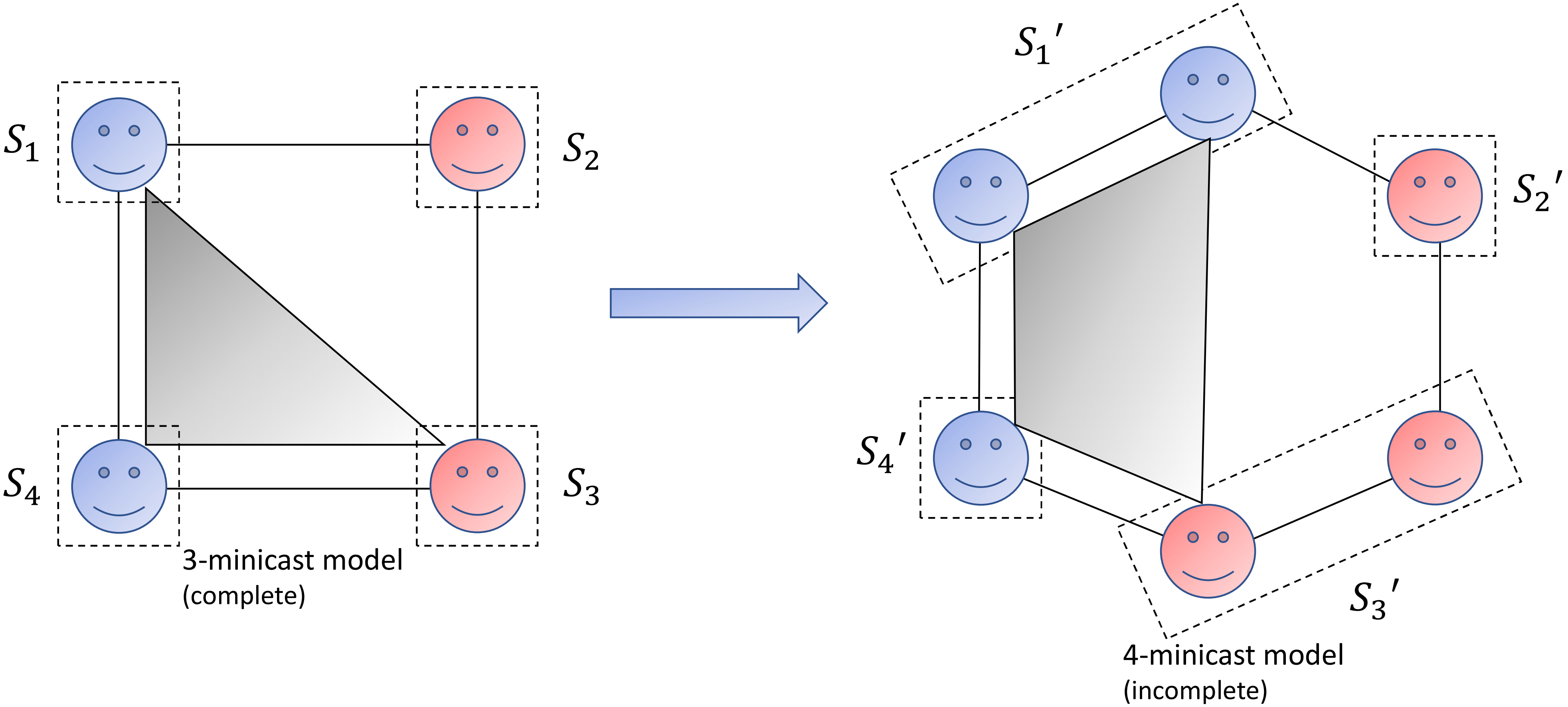

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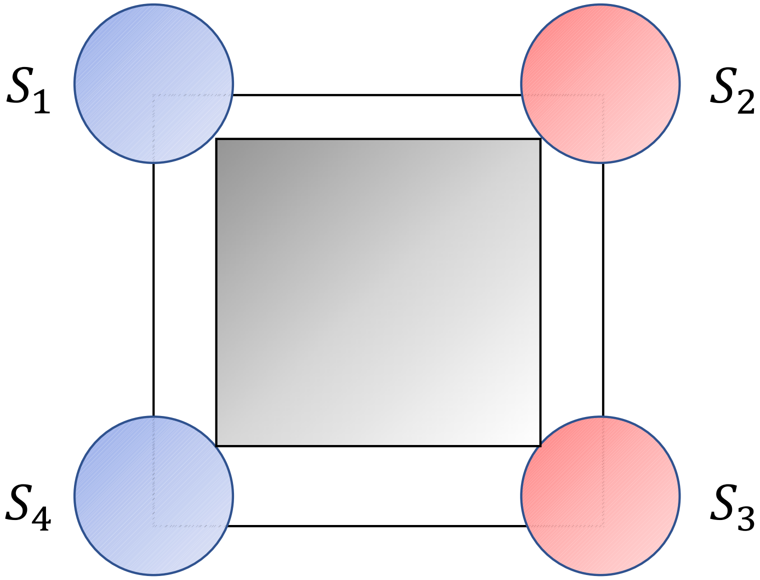

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- $n$ parties
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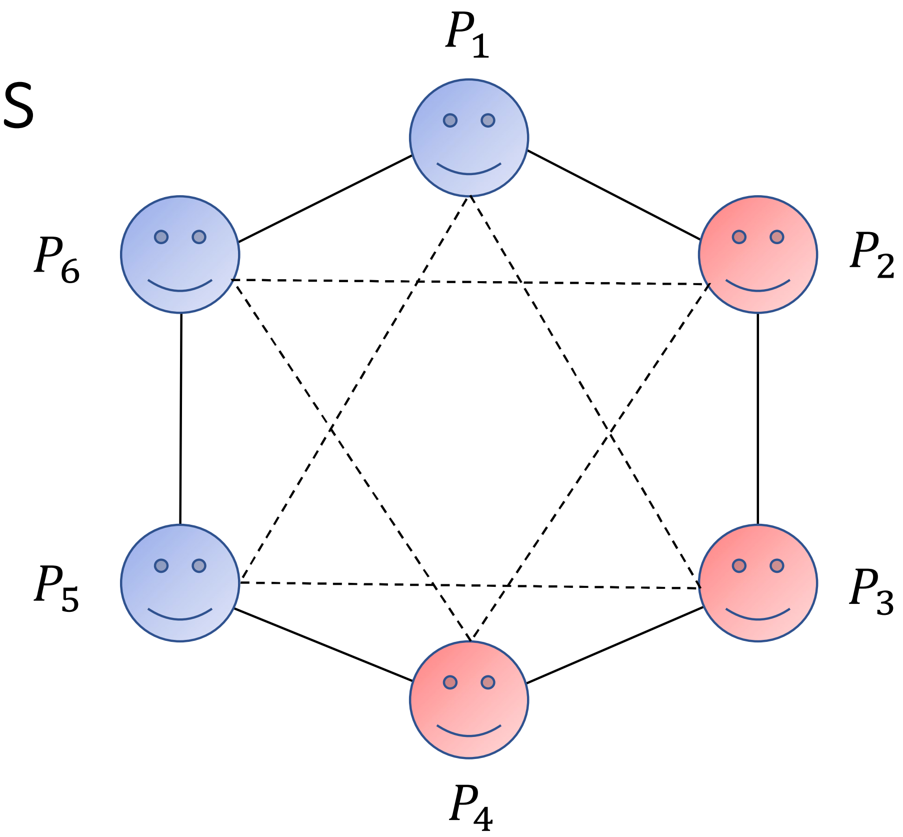
Idea: Simulate *missing/non-essential* $b$-minicast channels with *local* executions of [Ray15]’s protocol.

# Non-essential minicasts



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# Non-essential minicasts

- $b$ parties
- adv. is $b$-chain free
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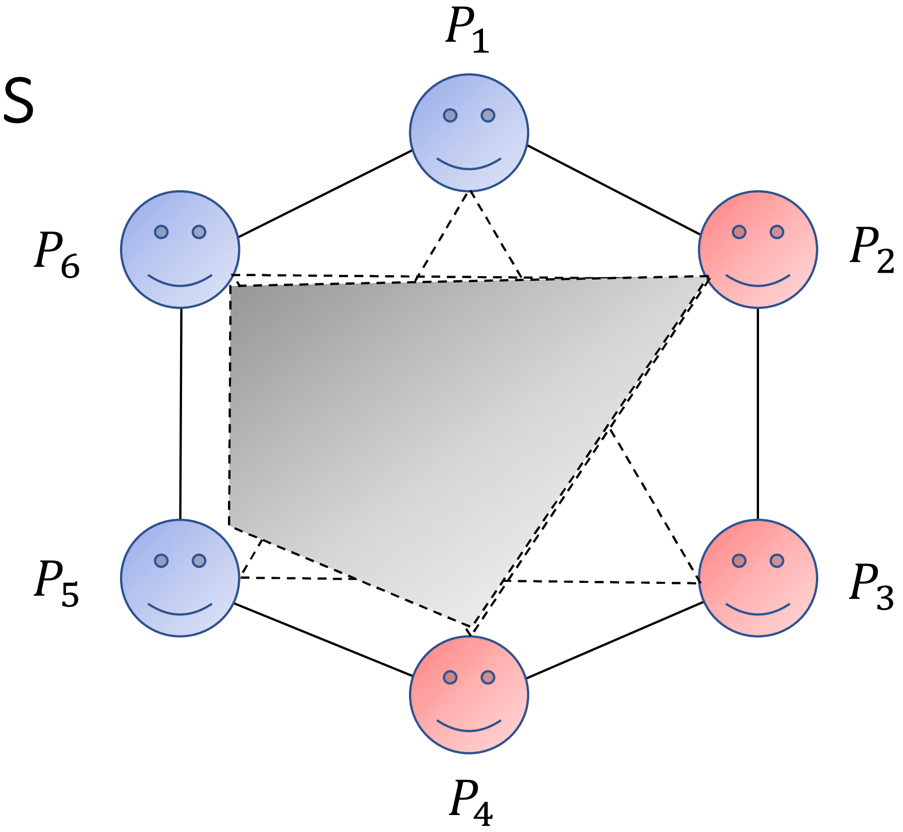
- Broadcast is possible [Ray15]

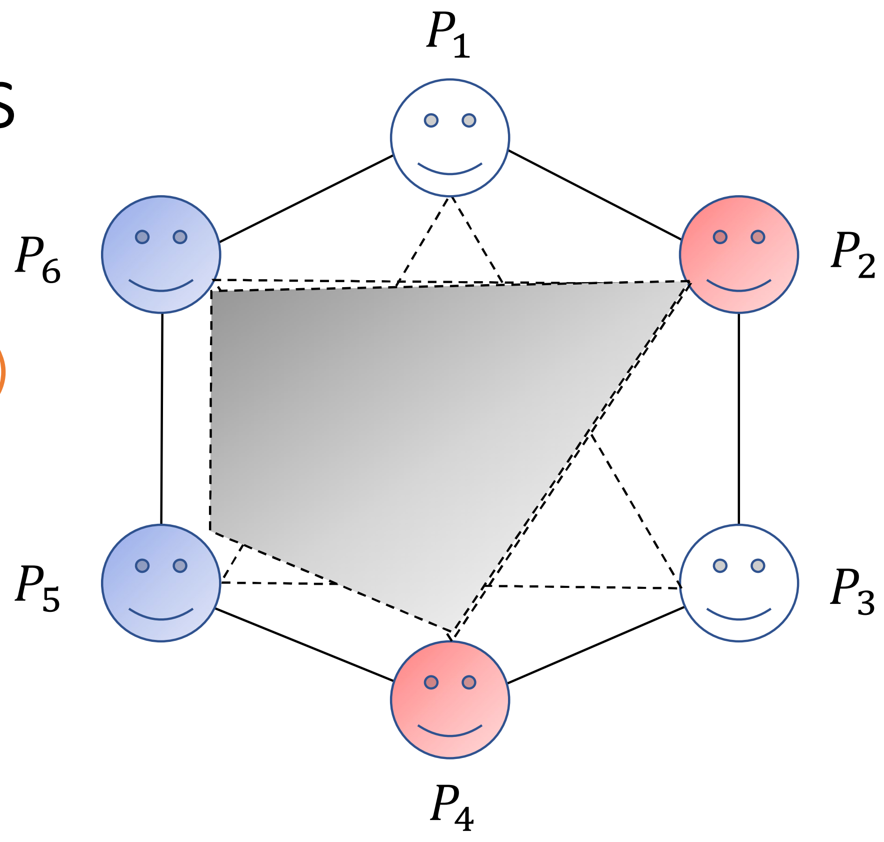


Idea: Simulate *non-essential* $b$-minicast channels with *local* executions of [Ray15]'s protocol.

# Non-essential minicasts

- $b$ parties
- adv. is $b$-chain free (locally restricted)
- $(b-1)$-minicast model (complete)

- Broadcast is possible [Ray15]



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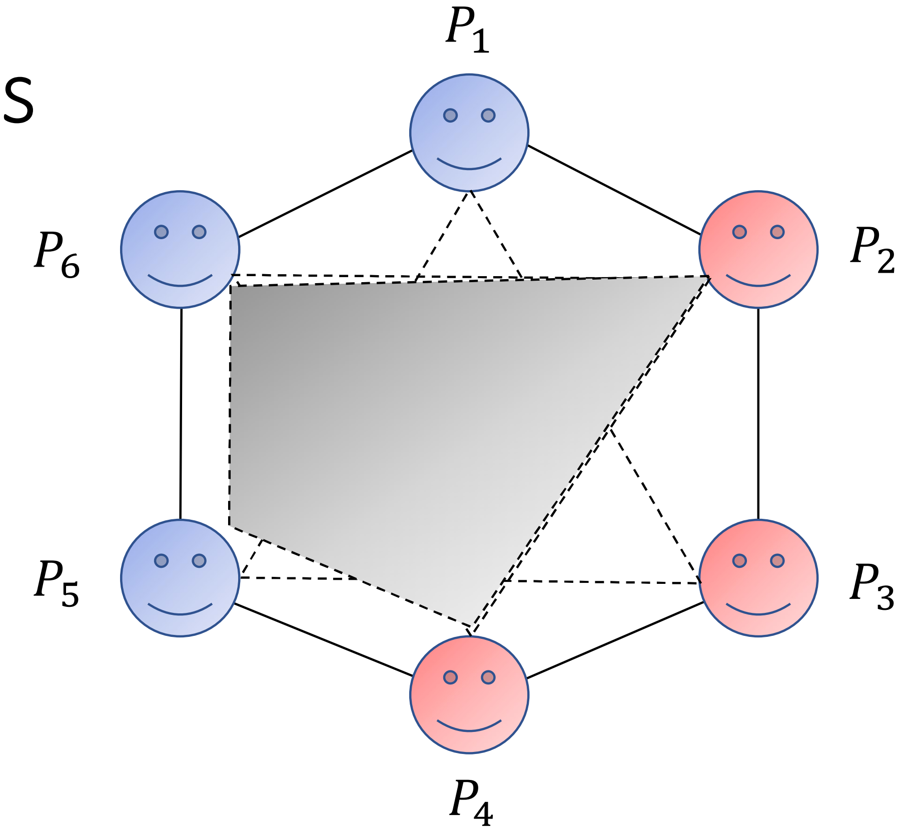

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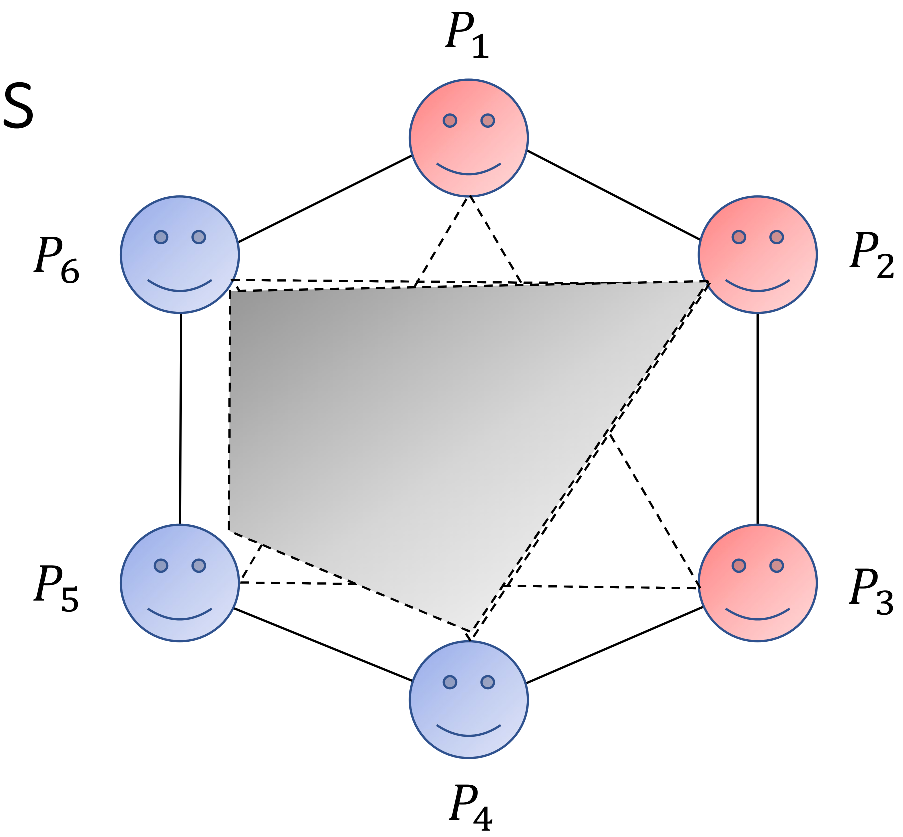

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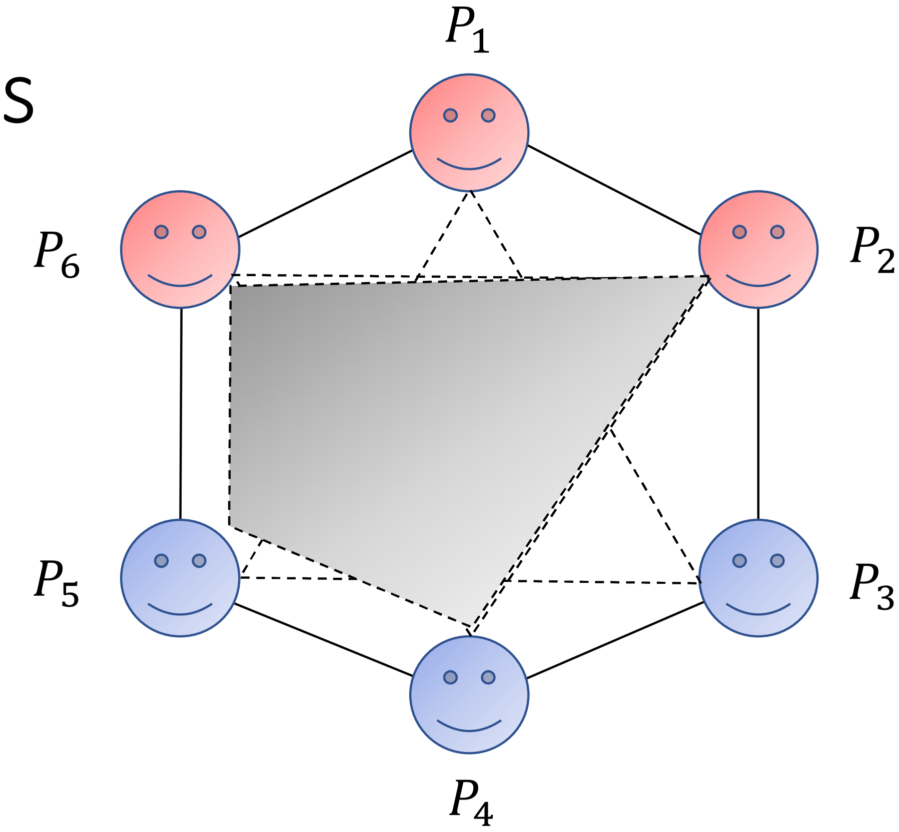

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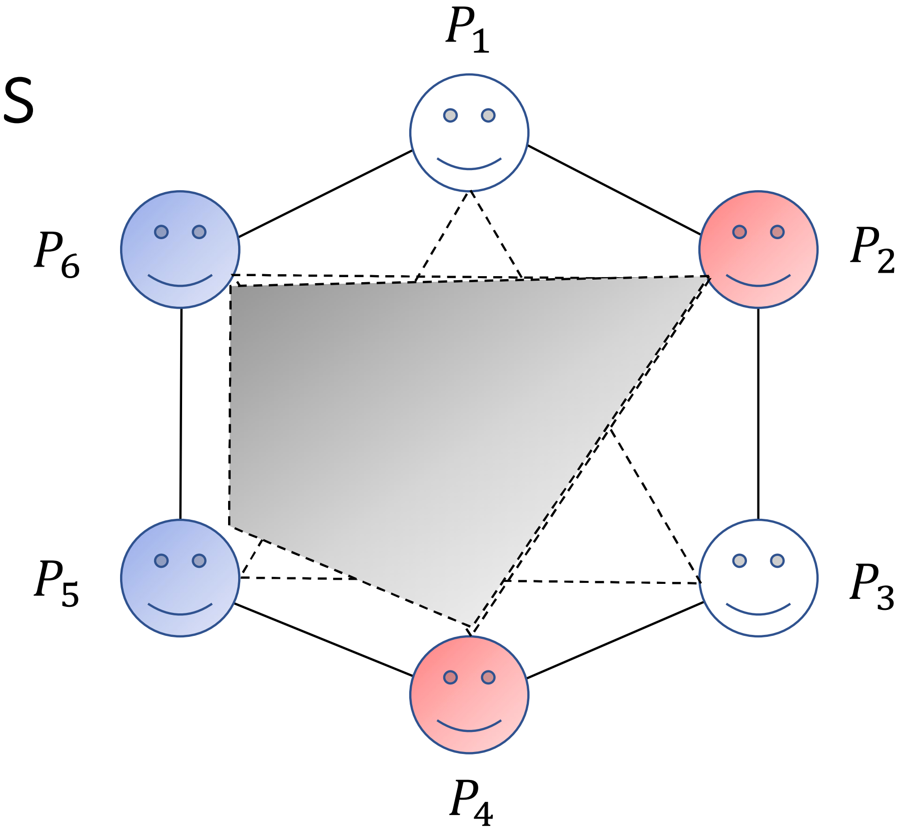

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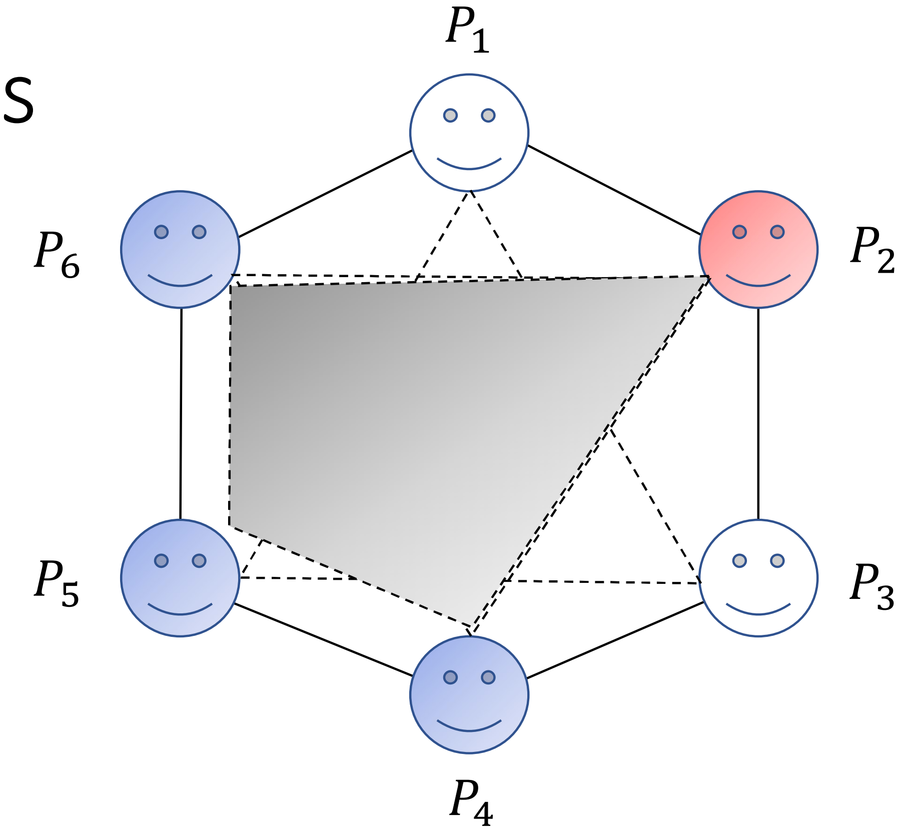

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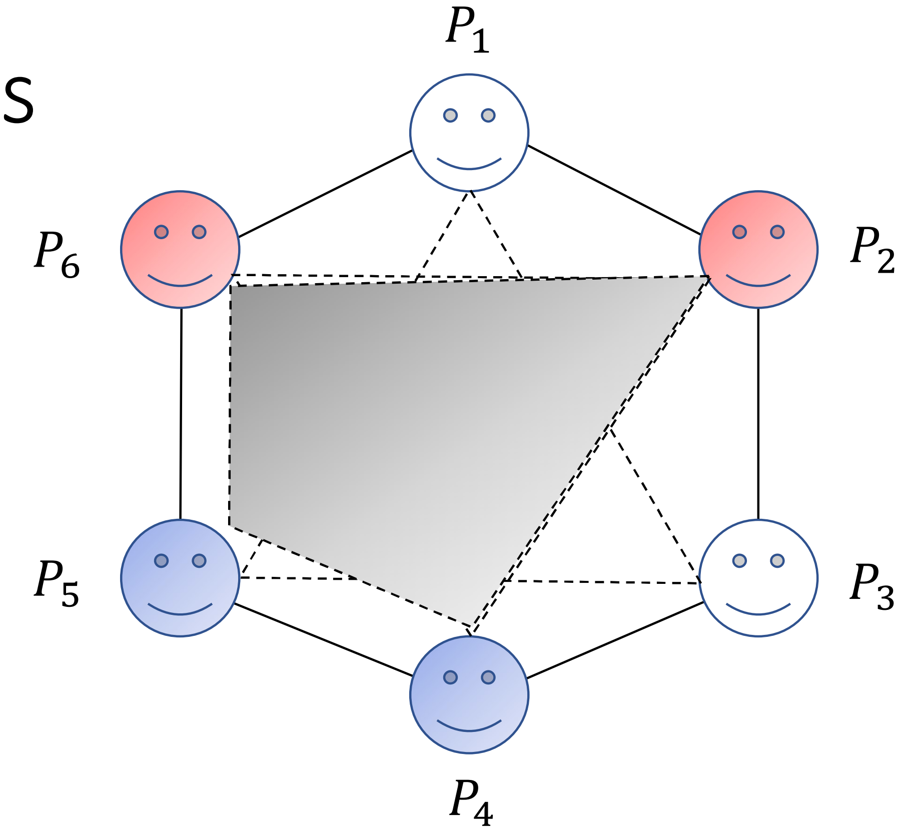

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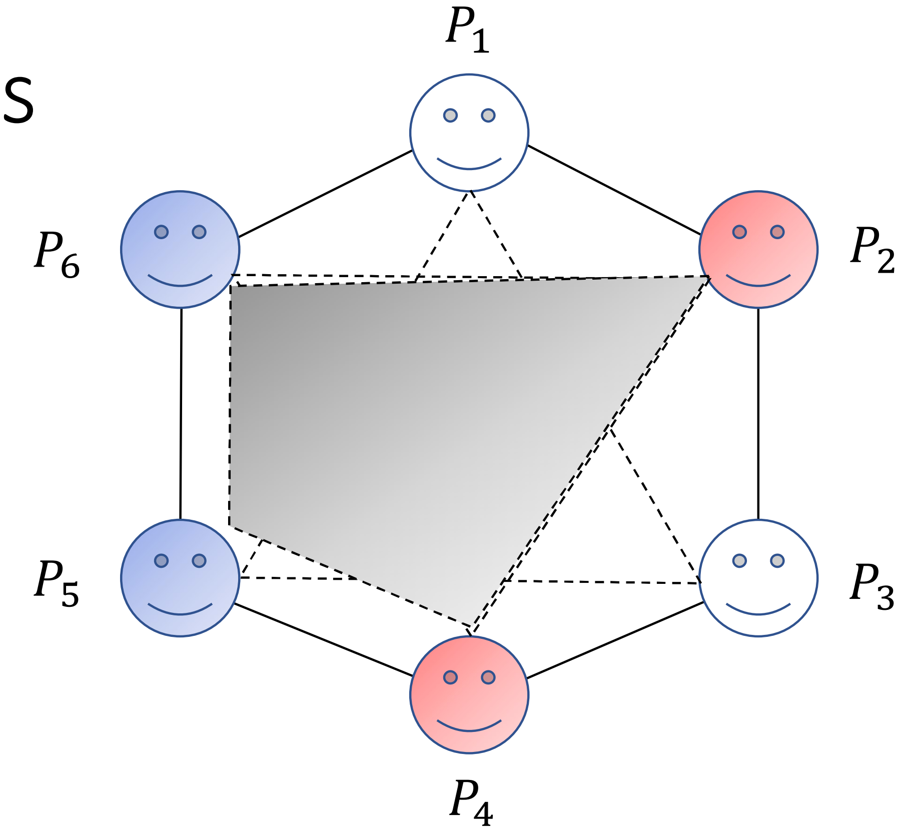
# Non-essential minicasts

- $b$ parties
- The projected adv. is $b$-chain free
- $(b-1)$-minicast model (complete)

- Broadcast is possible [Ray15]



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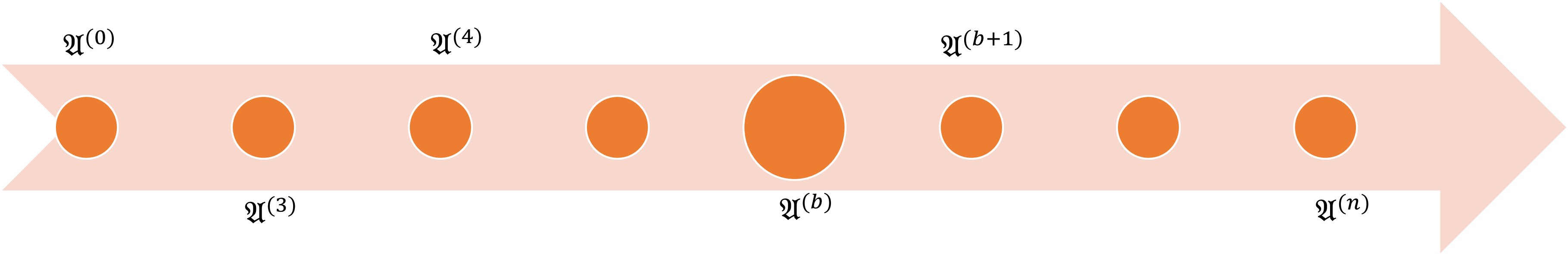
Condition *non-trivial* for certain weak class of adversaries in $\mathfrak{A}^{(b)}$, namely $b$-chain adversaries.

# Chain adversaries



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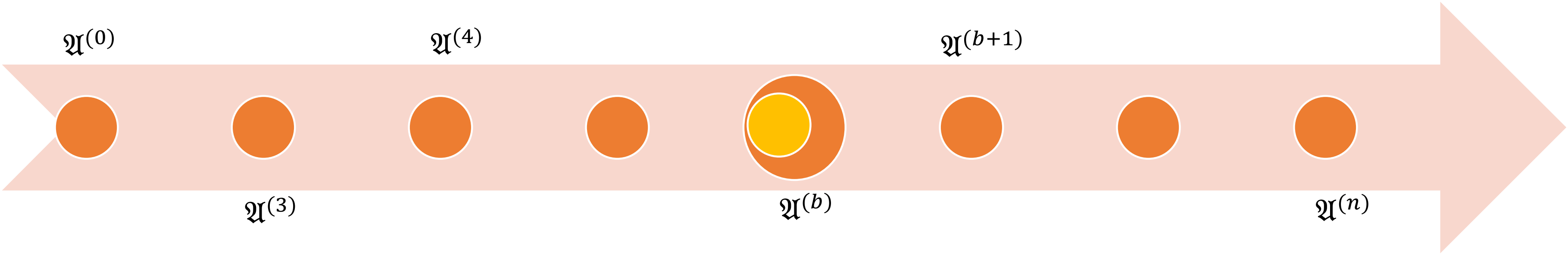

$\left(A \in \mathfrak{A}^{(b)}\right)$: $A$ contains $b$-chain(s) and $A$ is $(b+1)$-chain free

A $b$-chain adversary *just* contains a (single) $b$-chain, and nothing more

# Chain adversaries

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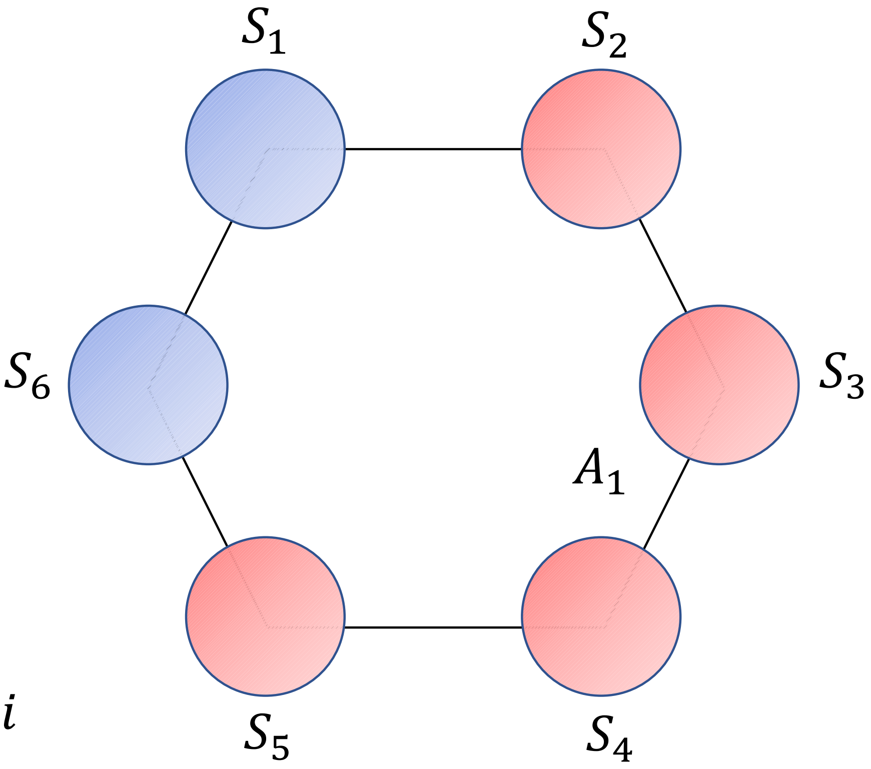

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- *b*-chain adversary corresponding to $S$:

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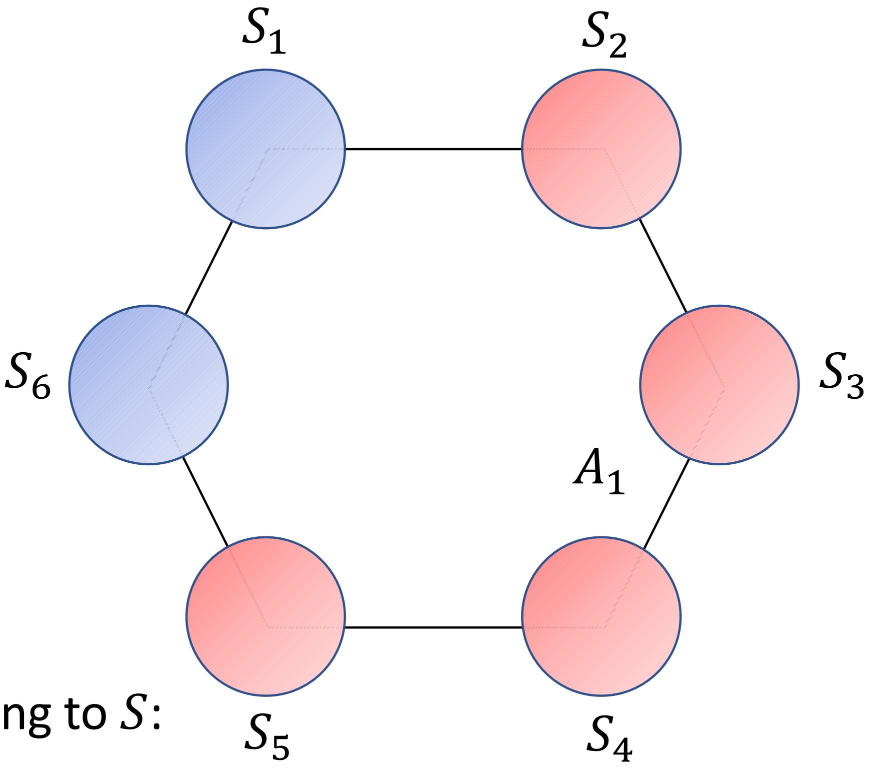

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  - there exist subsets of parties $\rho$ ($|\rho| = b$) such that $A^S[\rho]$ is $b$-chain free (i.e., $b$-minicast channel among $\rho$ is *non-essential*). [LMM20]

# Other results

- Our conditions allow us to derive bounds on the no. of $b$-minicast channels that are necessary and that suffice in achieving global broadcast in general networks secure against general adversaries.

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  - Thereby providing a way to extend [JMS12]’s quantitative analysis in general 3-minicast networks to higher $b$-minicast networks.

# Open problems

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- Implications of such results on broadcast in general $b$-minicast networks, secure against general adversaries, in a realistic setting.